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BIOMETRIKA.

EDITORIAL.

(I.) *The Scope of Biometrika.*

IT is intended that *Biometrika* shall serve as a means not only of collecting under one title biological data of a kind not systematically collected or published in any other periodical, but also of spreading a knowledge of such statistical theory as may be requisite for their scientific treatment.

A very few years ago, all those problems which depend for their solution on a study of the differences between individual members of a race or species, were neglected by most biologists. The complexity of organic structure is so great, and the number of distinguishable forms so enormous, that morphologists were obliged to simplify their conceptions by constructing for every species an ideal type, to which the individuals composing it conform with more or less exactness, and to neglect those deviations from the type which actually occur. Such simplification was not only justifiable, but absolutely necessary for many purposes; it has rendered enormous service to biology in the past, it does so still, and will continue to do so; nevertheless, there are many problems which cannot be dealt with by its aid.

The starting point of Darwin's theory of evolution is precisely the existence of those differences between individual members of a race or species which morphologists for the most part rightly neglect. The first condition necessary, in order that any process of Natural Selection may begin among a race, or species, is the existence of differences among its members; and the first step in an enquiry into the possible effect of a selective process upon any character of a race must be an estimate of the frequency with which individuals, exhibiting any given degree of abnormality with respect to that character, occur. The unit, with which such an

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enquiry must deal, is not an individual but a race, or a statistically representative sample of a race; and the result must take the form of a numerical statement, showing the relative frequency with which the various kinds of individuals composing the race occur.

As it is with the fundamental phenomenon of variation, so it is with heredity and with selection. The statement that certain characters are selectively eliminated from a race can be demonstrated only by showing statistically that the individuals which exhibit that character die earlier, or produce fewer offspring, than their fellows; while the phenomena of inheritance are only by slow degrees being rendered capable of expression in an intelligible form as numerical statements of the relation between parent and offspring, based upon statistical examination of large series of cases, are gradually accumulated.

These, and many other problems, involve the collection of statistical data on a large scale. That such data may be rendered intelligible to the mind, it is necessary to find some way of expressing them by a formula, the meaning of which can be readily understood, while its simplicity makes it easy to remember. The recent development of statistical theory, dealing with biological data on the lines suggested by Mr Francis Galton, has rendered it possible to deal with statistical data of very various kinds in a simple and intelligible way, and the results already achieved permit the hope that simple formulæ, capable of still wider application, may soon be found.

The number of biologists interested in these questions, and willing to undertake laborious statistical enquiries, is already considerable, and is increasing. It seems, therefore, that a useful purpose will be served by a journal especially devoted to the publication of statistical data, and of papers dealing with statistical theory. Many persons are deterred from the collection of biometric data, by the difficulty of finding such a means of publishing their results as this journal will afford, and those results which are published frequently lose much of their value because the data on which they are based are withheld, or because they are isolated in publications largely devoted to other forms of investigation. Further, *Biometrika* will endeavour to introduce a uniformity of statistical treatment, terminology, and notation, so that results obtained by different investigators on different types of life may be easily and effectively compared.

Biometrika will include (*a*) memoirs on variation, inheritance, and selection in Animals and Plants, based upon the examination of statistically large numbers of specimens (this will of course include statistical investigations in anthropometry); (*b*) those developments of statistical theory which are applicable to biological problems; (*c*) numerical tables and graphical solutions tending to reduce the labour of statistical arithmetic; (*d*) abstracts of memoirs, dealing with these subjects, which are published elsewhere; and (*e*) notes on current biometric work and unsolved problems. It is proposed to include memoirs written in English, German, French, or Italian.

(II.) *The Spirit of Biometrika.*

It is almost impossible to study any type of life without being impressed by the small importance of the individual. In most cases the number of individuals is enormous, they are spread over wide areas, and have existed through long periods. Evolution must depend upon substantial changes in considerable numbers and its theory therefore belongs to that class of phenomena which statisticians have grown accustomed to refer to as *mass-phenomena.* A single individual may have a variation which fits it to survive, but unless that variation appears in many individuals, or unless that individual increases and multiplies without loss of the useful variation up to comparatively great numbers—shortly, until the fit type of life becomes a mass-phenomenon, it cannot be an effective factor in evolution. The moment this point is grasped, then whether we hold variation to be continuous or discontinuous in magnitude, to be slow or sudden in time, we recognise that the problem of evolution is a problem in statistics, in the vital statistics of populations. Whatever views we hold on selection, inheritance, or fertility, we must ultimately turn to the mathematics of large numbers, to the theory of mass-phenomena, to interpret safely our observations. As we cannot follow the growth of nations without statistics of birth, death, duration of life, marriage and fertility, so it is impossible to follow the changes of any type of life without its vital statistics. The evolutionist has to become in the widest sense of the words a registrar-general for all forms of life. When he cannot observe and measure in Nature, then he must experiment on "populations" within the laboratory. But few biological laboratories have the space or the resources needed for dealing with the vital changes of populations, still less do the means at the disposal of individuals suffice for carrying out extensive experiments of this character. Much has been done and undoubtedly more will be done by the Marine Biological Laboratories for the study of mass-phenomena, but what is urgently needed is the establishment of a well-equipped Biometric Farm Laboratory, where breeding and survival experiments on large numbers could be carried out with ample room and care and, when necessary, for long periods. To this point we hope to return, and we shall not cease to urge its importance*.

But if we have thus to deal with a mass-phenomenon, may we not ask how it came about that the founder of our modern theory of descent made so little appeal to statistics? An illustration may aid us; the structure of our present theory of the moon is the creation of Newton, using his characteristic geometrical methods. But the practical astronomer in all lunar investigations to-day applies the analytical methods subsequently devised by the French mathematicians. The characteristic bent of Charles Darwin's mind led him to establish the theory of descent

* The failure of an attempt in this direction made a few years ago was, we believe, largely due to the fact that some of its supporters had not realised that the problem of evolution is a problem in the vital statistics of large numbers.

without mathematical conceptions; even so Faraday's mind worked in the case of electro-magnetism. But as every idea of Faraday allows of mathematical definition, and demands mathematical analysis in its modern statement, so every idea of Darwin—variation, natural selection, sexual selection, inheritance, prepotency, reversion—seems at once to fit itself to mathematical definition and to demand statistical analysis. Nor was the statistical conception itself entirely wanting in Darwin's work. *The Cross and Self-Fertilisation of Plants* forms a splendid collection of statistical observations and experiments which offers many points of departure for further statistical research*. That Darwin's mind did not work easily in mathematical lines is, perhaps, best evidenced in the passage of a letter of 1857 to Sir John Lubbock written when Darwin was dealing with the statistics of varieties in species as deduced from *Floras*:

> You have done me the greatest possible service in helping me to clarify my brains. If I am as muzzy on all subjects as I am on proportion and chance,—what a book I shall produce! (*Life*, II. p. 104).

But that he realised the importance of the statistical method for his investigations is evidenced not only by this very passage, but by several others. Thus considering the *variation* of our common species he writes in 1846:

> Andrew Smith once declared he would get some hundreds of specimens of larks and sparrows from all parts of Great Britain, and see whether, with finest measurements, he could detect any proportional variations in beaks or limbs, etc. This point interests me from having lately been skimming over the absurdly opposite conclusions of Gloger and Brehm (*Life*, II. p. 35).

Andrew Smith indeed missed the opportunity of being a veritable biometric pioneer!

Elsewhere Darwin recognises the importance of determining the variability of skeletons by measuring limbs (*Life*, II. p. 50). But, perhaps, the strongest evidence of his consciousness that biometry offers the only possible solution of problems of *inheritance* occurs in the words:

> I write now to say that I have been looking at some of our mongrel chickens, and I should say *one week old* would do very well. The chief point which I am, and have been for years, very curious about, is to ascertain whether the *young* of our domestic breeds differ as much from each other as do their parents, and I have no faith in anything short of actual measurement and the Rule of Three (*Life*, II. p. 51).

These words prove fully Darwin's consciousness not only of the need of measurements, but also of arithmetical work upon such data in the case of heredity. They may well serve as a motto for *Biometrika* and for all biometricians: *I have no faith in anything short of actual measurement and the Rule of Three.*

It is not a mere formal clothing of biological conceptions with mathematical symbols that is here indicated, or that we are considering, when we say that all

* See for example Mr G. U. Yule's use of Darwin's data in his recent memoir "On the Association of Attributes in Statistics," *Phil. Trans.* Vol. 194, A., 258.

Darwin's ideas fit themselves to algebraic definition. On the contrary—exactly as in the like case of the mathematical treatment of Faraday's conceptions of electro-magnetism—the symbolic analysis widens our notions, it leads us at once to new points of view and it directly suggests—perhaps this is its most important advantage—fresh points for observation and novel directions for experimental research.

The danger will no doubt arise in this new branch of science that—exactly as in some branches of physics—mathematics may tend to diverge too widely from Nature. The biologist, the mathematician and the statistician have hitherto had widely differentiated fields of work. Each one of these fields is full of pitfalls, and when the worker amid living types wanders among symbolic forms, the mathematician by profession must give him a helping hand if he stumbles over a determinant or gets entangled in a differential. A like patience must be extended by the biologist to the mathematician when he makes blunders at which the morphological tyro would smile. As Mr Francis Galton said a few years ago, for these new problems we want a scientific firm with a biologist and a mathematician as acting partners and a logician as a consulting partner. Patient endeavour to understand each other's methods, and to bring them into harmony for united ends and common profit—this is the only method by which we can win for biometry a recognised place in the world of science and in the accepted academic curricula of the universities. The day will come—is, perhaps, already dawning with the younger workers—when we shall find mathematicians who are competent biologists, and biologists who are competent mathematicians, but our universities both as to teachers and laboratories are not yet adapted for the training of such men, and for some time to come we can in the main only hope for effective partnership and not for the all-round biometrician. We have a splendid, almost untraversed field to work in, and a great task to perform in winning not only full recognition from the scientific world, but public support for our work. If these conditions are kept in view the diverse degrees of mathematical and biological knowledge exhibited in our pages will not oppress our readers. We shall publish careful biometric observations, even if they be accompanied by only the most elementary statistical treatment; we shall look forward to our mathematical workers supplementing such fundamental observations by more elaborate statistical calculations. For this reason we shall not only print as copious observational and experimental data as possible, but endeavour to form a manuscript collection of such data available for further research. We hope that every number of *Biometrika* will present statistical material ready for the mathematician to calculate and to reason upon. All such investigations ancillary to data appearing in our pages we shall receive gladly and publish at the earliest opportunity.

On the other hand the biologist will find in our pages algebraic analysis which may repel him. We would still ask his attention for the general conclusions and for the formulæ reached by the mathematician. The biologist will find that they frequently suggest observations and experiments which he alone is in a position to undertake satisfactorily. We shall aid the more arithmetical part of his work by

diagrams and numerical tables wherever it seems possible. In this manner we hope that *Biometrika* will provide material for both branches of science; that it will not only publish valuable biometric and statistical researches, but serve as a store-house of unsolved problems for both unemployed biologist and mathematician. We trust that bringing these men together may widen the activity of both.

Many of the problems of biometry can only be approached from the standpoint of the cooperative collection of material and reduction of statistics, and in this respect we shall strive to form a link between scattered biometric workers. *Biometrika* will ask for aid in cooperative work, and be at all times ready to publish requests for aid, and the forms and schedules which our contributors desire to be circulated or filled in. In this way it may be hoped that a guild of qualified collectors and workers may be gradually formed to whom appeal may be made for collecting, counting and observing. There are many men and women with the necessary training, scattered about this and other countries, who without having the opportunity for initiating original work are not only competent but glad to assist with collecting-box, camera or pencil. From such workers the Editors will be glad to hear and will endeavour to put them in touch with those desiring their aid.

Extensions, corrections, criticisms of the results published in our pages we shall heartily welcome whatever be their source. We expect to receive stalwart blows as well as to give them. All we shall demand in this respect is the chivalry which is needful in scientific controversy, which while combating error does not discourage honest endeavour. The most fertile men of science have made blunders, and their consciousness of such slips has been retribution enough; it is only their more sterile critics who delight to dwell too often and too long on such mistakes. In science, both in symbolic analysis and in our knowledge of Nature, we are very ignorant; we do not pretend that biometry will revolutionise our ideas of life. All we claim is that in certain aspects of biological research, biometry is an instrument which can aid us effectively in our gropings after truth. Only let the spirit in which it is used be that of the master-mind, the ideal so well and faithfully portrayed in the form and features on our frontispiece. *Ignoramus; in hoc signo laboremus!*

BIOMETRY.

By FRANCIS GALTON.

This Journal is especially intended for those who are interested in the application to biology of the modern methods of statistics. Those methods deal comprehensively with entire species, and with entire groups of influences, just as if they were single entities, and express the relations between them in an equally compendious manner. They commence by marshalling the values in order of magnitude from the smallest up to the largest, thereby converting a mob into an orderly array, which like a regiment thenceforth becomes a tactical unit. Those to whom these considerations are new, will grasp the results more easily by thinking of the array in its simplest, though not necessarily in its most convenient, form for mathematical treatment. Let them conceive each value to be represented by an extremely slender rod of proportionate length, and the rods to be erected side by side, touching one another, upon a horizontal base. The array of closely packed rods will then form a plane area, bounded by straight lines at its sides and along its base, but by a flowing curve above, which takes note of *every one* of the values on which it is founded, however immense their multitude may be. The shape of the curve is characteristic of the particular group of values to which it refers, but all arrays have a family resemblance due to similarity of origin; they all drop steeply at one end, rise steeply at the other, and have a sloping back. An array that has been drilled into some such formation as this, is the tactical unit of the new statistics. Its outline is expressed by a general formula whose constants are adapted to each particular case, and, being thus brought within the grip of mathematics, the internal relations of an array and their relations to those of any other array can be expressed in exact numerical forms. The new methods occupy an altogether higher plane than that in which ordinary statistics and simple averages move and have their being. Unfortunately the ideas of which they treat, and still more the many technical phrases employed in them, are as yet unfamiliar. The arithmetic they require is laborious, and the mathematical investigations on which the arithmetic rests are difficult reading even for experts; moreover they are voluminous in amount and still growing in bulk. Consequently this new departure in science makes its appearance under conditions

that are unfavourable to its speedy recognition, and those who labour in it must abide for some time in patience before they can receive much sympathy from the outside world. It is astonishing to witness how long a time may elapse before new ideas are correctly established in the popular mind, however simple they may be in themselves. The slowness with which Darwin's fundamental idea of natural selection became assimilated by scientists generally, is a striking example of the density of human wits. Now that it is grown to be a familiar phrase, it seems impossible that difficulty should ever have been felt in taking in its meaning. But it was far otherwise, for misunderstandings and misrepresentations among writers of all classes abounded during many years, and even at the present day occasional survivals of the early stage of non-comprehension make an unexpected appearance. It is therefore important that the workers in this new field who are scattered widely through many countries, should close their ranks for the sake of mutual encouragement and support. They want an up-to-date knowledge of what has been done, and is doing, in it. They seek for opportunities of receiving judicious help from one another, sometimes in circulating questions, sometimes in discussing the preliminaries of new plans of campaign. Immense labour has too often been wasted in statistical research through a mistaken judgement of the value and real significance of the data employed. The fresh opinion of skilled onlookers is the safest test of the value of materials and affords a ready means of obtaining timely warning of the presence of vitiating conditions before an inquirer commits himself to any new statistical enterprise. Every investigator stands in need of expert criticism, for no pursuit runs between so many pitfalls and unseen traps as that of statistics.

This Journal, it is hoped, will justify its existence by supplying these requirements either directly or indirectly. I hope moreover that some means may be found, through its efforts, of forming a manuscript library of original data. Experience has shown the advantage of occasionally rediscussing statistical conclusions, by starting from the same documents as their author. I have begun to think that no one ought to publish biometric results, without lodging a well arranged and well bound manuscript copy of all his data, in some place where it should be accessible, under reasonable restrictions, to those who desire to verify his work. But this by the way. There remains another cogent reason of a very practical kind for the establishment of this Journal, namely that no periodical exists in which space could be allowed for the many biometric memoirs that call for publication. Biometry has indeed many points in common with Mathematics, Anthropology, Zoology, Botany, and Economic Statistics, but it falls only partially into each of these. An editor of any special journal may well shrink from the idea of displacing matter which he knows would interest his readers, in order to make room for communications that could only interest or even be understood by a very few of them. I am tempted to illustrate, or rather to over-illustrate, the coldness of welcome often afforded to a new departure in science, by an anecdote concerning the cause that really led to the foundation of the Geological Society

of London. I have rarely related it in conversation, fearing to give pain to some one, and I have never done so in print; neither can I find that any version of it has been published by others. But now that nearly a century has slipped past since the event, there can be no harm in digging up and bringing to light a buried but amusing historical fact.

The story was told me long, long ago, in the 'forties, by Mr George Bellas Greenough, F.R.S. I was then an eager youth fresh from college, and he an elderly man; it was as follows. In 1806–7, when Geology was in its infancy and travellers were scarce owing to European wars, Mr Greenough and a few young friends compiled a list of questions with the view of ascertaining how far the facts of Nature might agree with the competing geological theories current in those days. Sir Joseph Banks was the President of the Royal Society at that time, an office which he exercised despotically for 43 years (1777–1820), becoming almost an autocrat over English scientific men. So it was to him that Mr Greenough and his young friends naturally went. They brought their questions and begged that copies of them might be circulated under official sanction among suitable persons, including foreign correspondents of the Royal Society. Sir Joseph was sometimes gracious in mood, frequently the reverse, and on this occasion he might be described as bearish. Not content with an emphatic "no," he dismissed them with words to the effect (in almost those very words, if my memory does not deceive me) that a few fools could ask more questions in half an hour than wise men might answer in years. The deputation departed, ready to burst with suppressed fury, and the moment they were quit of the house, agreed to circulate the questions on their own responsibility, which considering the persons and circumstances was an act of rare audacity. Out of this impromptu coalition, aided by a multitude of elsewhere recorded circumstances, the Geological Society was evolved, with Mr Greenough as its first President. (The official account of its origin is judiciously reticent, but not inconsistent with this little piece of history. It will be found in the preface to the first volume of its *Transactions*, published in 1811.) It is not in the least my intention to insinuate that Biometry might be served by any modern authority in so rough a fashion, but I offer the anecdote as forcible evidence that a new science cannot depend on a welcome from the followers of older ones, and to confirm the former conclusion that it is advisable to establish a special Journal for Biometry.

The primary object of Biometry is to afford material that shall be exact enough for the discovery of incipient changes in evolution which are too small to be otherwise apparent. The distribution of any given attribute, within any given species, at any given time, has to be determined, together with its relations to external influences. This affords a standard whence departures may be measured and the direction and rate of their progress ascertained. Evolutionary changes are exceedingly slow as a rule, but supposing that a thousand years or thirty generations of mankind, would suffice in some particular case for some conspicuous alteration in a species, exact measurements ought to discover its progress well within the limits

of a human lifetime. Moreover the forms by which distribution is expressed in the new method are excellently fitted to bring to light any survivals of a less advanced type, which may serve as evidence of recent change. Also they quickly indicate incipient changes, through their power of isolating aberrant forms, and then of measuring the degree in which any of these may be favoured by natural selection. The organic world as a whole is a perpetual flux of changing types. It is the business of Biometry to catch partial and momentary glimpses of it, whether in a living or in a fossil condition, and to record what it sees in an enduring manner. It is an after-process to combine those glimpses into a continuously changing scene, much as some tumultuous procession is made to live and move again by means of a "biograph." Each biometric investigation may be compared to a solitary boring in a level plain, whose underlying geology has to be ascertained. A comparison of the cores brought up, will supply evidence of the depths of each of the buried strata and will justify many interpolations of unseen portions between the borings. For instance, it may not require many investigations to establish statistical laws of heredity on a secure basis, by ascertaining the limits within which those that have been already observed may hold good in a moderate number of widely different types of plant and animal life. Biology could soon be raised to the status of a more exact science than it can as yet claim to be, if each of many biometricians would thoroughly work out his own particular plot, although those plots may be very far indeed from occupying the whole of the area that admits of being directly explored.

VARIATIONSSTATISTISCHE PROBLEME UND MATERIALIEN.

Von PROF. DR F. LUDWIG.

I.

Schon an anderem Ort* habe ich darauf hingewiesen, dass zwischen den zoologischen und botanischen Variationspolygonen ein wesentlicher Unterschied bestehen dürfte, insofern die letzteren vorwiegend komplex, polymorph, sind und nicht den einfachen Pearsonschen Typen monomorpher Variationspolygone entsprechen, wie sie von Anthropologen und Zoologen gefunden werden. Man kann das einmal mit dem bei der Pflanze *weniger begrenzten Wachsthum* erklären, bei dem *verschiedene* Entwicklungs-, bezüglich Wachsthumsstufen innerhalb der Species erreicht werden können (vgl. die Fibonaccikurven!); dann aber dürfte es zurückzuführen sein auf die bei den höheren *Pflanzen* im Gegensatz zu den höheren Tieren so *häufige asexuelle* und die ihr im Effect verwandte *autogame* sexuelle *Fortpflanzung*. Bei letzterer verhalten sich die Nachkommen wie Teile desselben Stockes, und sind nur im Besitz solcher Eigenschaften die das Mutterindividuum selbst kennzeichneten, die Variation führt zu den "petites espèces" deren Charaktere, wenn auch noch so minutiös, merkwürdig konstant gefunden werden. Die regelmässige sexuelle Fortpflanzung mit Xenogamie wird dagegen eine nivellierende Wirkung haben müssen, welche der Bildung "kleiner Arten" entgegensteht.

Diese Unterschiede in der Reihe der Fortpflanzungsarten, die beginnt mit den rein vegetativen Vermehrungsarten (bei dem europäischen *Kalmus*, bei *Ficaria verna, Trientalis europaea, Symphytum bulbosum, Apocynum hypericifolium, Lysimachia nummularia, Dielythra spectabilis*, der europäischen *Elodea canadensis* etc.†), mit der typisch parthenogenetischen Embryobildung (bei

* Ludwig; Een fondamenteel verschil in de veranderlijkheid bij het dier en de planten? *Kruidkundig Genootschap Dodonaea te Gent*, Elfde Jaargang, 1899, pp. 108—121.

† *Elodea* ist diöcisch, aber nur in dem einen Geschlecht in Europa verbreitet, die anderen genannten Arten sind selbststeril und bringen an Standorten, wo alle Individuen von demselben Stock abstammen, keine Samen. Vgl. F. Ludwig; On Self-sterility, *The Journal of the Royal Horticultural Society* (Hybrid Conference Report), Vol. xxiv. April 1900, pp. 214—217.

*Antennaria alpina**, *Alchemilla alpina, speciosa, alpestris*, u. a. *Alchemilla*arten†, die durch zahlreiche "petites espèces" ausgezeichnet sind), mit typischer Kleistogamie (bei *Plantago virginica* in Europa, *Collomia grandiflora* an vielen europäischen Standorten etc.), der Autogamie (wie bei *Erophila verna*, von der De Bary und Rosen zahlreiche "petites espèces" cultiviert haben), und endet mit der ausschliesslichen Xenogamie (mit Selbststerilität, ohne besondere asexuelle Vermehrung), sie müssen auch, wie ich meine, in den Variationspolygonen zum Ausdruck kommen.

Wo die sexuelle Fortpflanzung allein zur Geltung kommt, wird unter den von der Natur gegebenen Verhältnissen ein zwar komplexes aber *nahezu konstantes Variationspolygon* zu Stande kommen, wie ich es für *Chrysanthemum leucanthemum, Ch. inodorum, Ch. segetum* etc. nachgewiesen habe. Bei meinen "Hyperbinomialcurven" und verwandten Typen‡ haben die die abnorme Gipfelhöhe bedingenden nicht variierenden Exemplare wahrscheinlich vegetativen (asexuellen) oder autokarpen Ursprung. Da wo lokal die vegetative oder parthenogenetische (oder kleistogame) Fortpflanzung die ausschliessliche ist, werden die "petites espèces" ganz so wie in andern Fällen, wo die Individuen verschiedener Arten durch einander gezählt worden sind, *inkonstante Komplexcurven* (von zwar *gleicher Gipfellage, aber inkonstanter Frequenz*) ergeben, sofern nicht eine der kleinen Arten ganz beträchtlich überwiegt.

Das soeben erörterte scheint durch die folgenden Zählungen bei *Ficaria verna* Bestätigung zu finden. *Ficaria verna* (*Ranunculus Ficaria L.*) pflanzt sich sehr stark durch ober- und unterirdische Bulbillen fort und da sie selbststeril ist, so dass alle von demselben Individuum stammenden Bulbillen auch nach Generationen nur untereinander unfruchtbare Pflanzen ergeben, so bildet sie an vielen Orten überhaupt keine Früchte und pflanzt sich vorwiegend asexuell fort. Obwohl von den Systematikern nur wenig Varietäten beschrieben werden (vgl. Federico Delpino: *Dimorfismo del Ranunculus Ficaria L., Memoria letta alla R. Accademia delle Science dell' Istituto di Bologna* nella Sessione dell' 11 Aprile 1897, Bologna 1897, pp. 685—710), zeigen doch die folgenden statistischen Ergebnisse, dass dieselbe abgesehen von dem durch Delpino aufgefundenen Gynodimorphismus in manchen Gegenden (wo es kleinblütige ♀ und grossblütige ☿ gibt) in mehrere kleine Arten zu spalten ist.

Unsere Beobachtungen beziehen sich auf die Zahl der Blütenteile und zwar vorerst auf die relativen Anzahlen von Kelch- und Kronenblättern (erstere mit K, letztere mit C im Folgenden bezeichnet). (Ich teile hier das ganze Material

* *Cf.* H. O. Juel; *K. Svenska Vetenskaps-Academiens Handlingar*, Bd. xxxiii. No. 5, 59 pp. Stockholm 1900.

† Sv. Murbeck; *Lunds Univ. Arsskrift* xxxvi. 2, No. 7, 46 pp.; No. 9, 20 pp. Lund 1901.

‡ Vgl. Ludwig; Die pflanzlichen Variationskurven und die Gauss'sche Wahrscheinlichkeitskurve, *Bot. Centrbl.* Bd. 73, 1898.—G. Duncker; *Die Methode der Variationsstatistik*, Leipzig 1899, p. 127 ff.—G. Duncker; Variation und Asymmetrie bei *Pleuronectes flesus L.*, *Wissensch. Meeresuntersuch.* N. F. Bd. 3 H. 2, p. 364.

mit, ob wohl dasselbe in dem vorliegenden Aufsatz nur nach einer Richtung hin zur Discussion kommt.)

Korrelative Zählungen der Kelch- und Kronblätter von *Ficaria verna*.

A. *Greiz (Lehrerseminar) I^te Tausend.*

Kronblätter

Kelchblätter	4	5	6	7	8	9	10	11	12	13	14	
0				3	5		1					9
1					1		1					2
2					1	1						4
3		8	16	83	**270**	65	26	9	3	1	1	**482**
4	1	3	21	64	97	59	29	5	4		1	284
5		1	10	30	52	44	33	21	19	1		211
6			1		1	2	2			2		8
	1	12	48	180	**427**	171	92	37	26	4	2	1000

B. *Greiz (Lehrerseminar) II^te Tausend.*

Kronblätter

Kelchblätter	5	6	7	8	9	10	11	12	13	14	15	
0				3	1	1	1					6
1			1	3								4
2		1	4	3	3							11
3	2	11	76	**270**	89	19	2	3	1			**473**
4		11	59	86	72	56	15	9	3		1	312
5	1	14	22	61	31	30	17	9	3			188
6				2	2	1	1					6
	3	37	162	**428**	198	107	36	21	7		1	1000

C. *Greiz (Seminar und Gymnasium) III^te Tausend.*

Kronblätter

Kelchblätter	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	
0						2								2
1					1									1
2				4	4	6	3	2		1	1			21
3	1	1	4	7	78	**262**	107	48	6					**514**
4		1	1	10	43	80	62	50	15	9	2		1	274
5			1	3	24	45	29	40	17	3				162
6					2	5	4	3	5	4	1			24
7						1		1						2
	1	2	6	24	152	**401**	205	144	43	17	4		1	1000

Letzteres C. setzt sich zusammen aus 300 Zählungen der Gymnasiasten von 1900, 300 solchen von 1901 und 400 Zählungen der Seminaristen von 1901 wie folgt:

D. 300 *Gymnasiastenzählungen.* 1900.

Kronblätter

Kelchblätter	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	
2			1		4	2	2		1	1			11
3			6	20	**86**	31	20	5					**168**
4	1		5	10	20	14	15	2	6			1	74
5			2	6	10	4	12	1	2				37
6				1	1	1	1	1	4	1			10
7													
	1		14	37	**121**	52	50	9	13	2		1	300

E. 300 *Gymnasiastenzählungen.* 1901.

Kronblätter

Kelchblätter	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	
2			2		1				3
3	1	1	1	30	**64**	31	14	1	**143**
4		1	1	12	29	21	17	12	93
5		1		12	13	5	11	7	49
6				1	3	2	1	3	10
7					1		1		2
	1	3	4	56	**111**	59	45	24	300

F. 400 *Seminaristenzählungen.* 1901.

Kronblätter

Kelchblätter	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	
0						2						2
1					1							1
2				1	4	1	1					7
3	1		3		28	**112**	45	14				**203**
4				4	21	31	27	18	1	3	2	107
5				1	6	22	20	17	9	1		76
6						1	1	1	1			4
	1		3	6	60	**169**	94	50	11	4	2	400

G. *Greiz (Lehrer Winkler) IV^te Tausend.*

Kronblätter

Kelchblätter	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	
2												
3			2	10	56	**530**	89	23	7	1		**718**
4				3	32	56	26	10	6	3	1	137
5			4	9	9	49	26	19	11	1	8	136
6					1	1	4	1	1			8
7							1					1
			6	22	98	**636**	146	53	25	5	9	1000

H. *Gera* (*Realgymnasium*) 1000.

Kronblätter

Kelchblätter	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	
0					3								3
1													
2		3	3	2	7	4	1	1					21
3		12	22	67	**460**	117	33	12	7	2		1	**733**
4		1	5	35	55	35	13	6	2				152
5			1	8	36	19	13	4	2	3			86
6					2		2	1					5
		16	31	112	**563**	175	62	29	11	5		1	1000

I. *Gera* (*Realgymnasium*) 675 *Zählungen.*

Kronblätter

Kelchblätter	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	
2						1					1
3		2	15	73	**347**	45	36	8			**526**
4		2	3	18	54	10	11	3			101
5		5	5	7	9	12	5				43
6				1			3				4
		9	23	99	**410**	68	55	11			675

K. *Gera* (*Lehrer Auerbach und einige Schülerzählungen*) 712 *Zählungen.*

Kronblätter

Kelchblätter	*7*	*8*	9	10	
2	4				4
3	5	**498**	93	15	**611**
4		14	11	3	28
5		37	8	24	69
	9	**549**	112	42	712

L. *Trogen (Kt. Appenzell, Schweiz)* 1900, *Kantonschule (A. Heyer)*, 285 *Zählungen.*

Kronblätter

Kelchblätter	5	6	7	8	9	10	11	
3	1	2	11	**133**	32	13	2	**194**
4		1	18	42	10	2	1	74
5		1	4	8	1	1		15
6			1			1		2
	1	4	34	**183**	43	17	3	285

Die Einzelzählungen ergeben, und das kommt auch in den vorstehenden Gesammtergebnissen zum Ausdruck, dass in Deutschland (Greiz, Gera) und der Schweiz (Trogen) ganz bedeutend überwiegt *als Normalform* K_3C_8 (mit 3 Kelchblättern und 8 Kronblättern), dass aber *in verschiedener Präponderanz*— und das beweist eben ihre Bedeutung als "petites espèces"—verbreitet sind *namentlich die Formen* K_5C_8, K_5C_{10}, K_4C_{10}, K_3C_5, K_5C_5, K_3C_{13}, K_5C_{13}, und—selten K_6C_8.

Die Zählungen A—G sind von Schülern verschiedener höheren Lehranstalten in der Umgebung von Greiz an den verschiedensten Standorten gemacht worden und zwar "ohne Wahl." Bemerkenswert sind die unter A, B und F aufgeführten Zählungsergebnisse der Zöglinge des Greizer Lehrerseminars die von 41 Schülern gemacht wurden. Das erste Tausend (A) wurde von den Herren Seminaroberlehrern Collmann und Lotter, welche die Zählungen leiteten, zusammengestellt (von 18 Schülern), aus dem übrigen Material wählte ich beliebige 1000 Zählungen (sie stammten von 17 anderen Schülern) für B und den Rest von 400 Zählungen (von 6 Schülern) für F aus. Trotz der verschiedener Zähler und des verschiedenen Ursprungs erwiesen sich die Resultate für K_3, C_8, K_3C_8 in hohem Grad übereinstimmend: für

	K_3,	C_8,	K_3C_8
bei A	48·2%	42·7%	27·0%
B	47·3%	42·8%	27·0%
(F	50·7%	42·3%	28·0%).

Die Zählungen waren im Anfang der Blütezeit gemacht; die von Herrn Lehrer Winkler (G) von gleichen Orten meist zu Ende derselben. Erfahrungsgemäss findet man zuletzt höhere Zahlen, dies spricht sich auch in den Ergebnissen bei G bezüglich der Blumenblätter aus. Andere Unterschiede ergeben sich daraus, dass gegen Ende der Blütezeit die Blätter leicht ausfallen und Herr W. nur vollkommene Blüten berücksichtigend wohl etwas über das Ziel hinaus kam.

Hier ist K_3C_8 bei 53% der Zählungen vorhanden, bei den Zählungen der Gymnasiasten ergeben sich 1901 etwas andere Zahlen als 1900 nämlich für

K_3C_8: 1900 28·7%; 1901 21·3%.

Die Resultate unter D, E, F ergeben zusammen unter C für

K_3,	C_8,	K_3C_8
51·1%	40·1%	26·2%

(mit Rücksicht auf die etwas verschiedene Zeit wieder gut mit A, B übereinstimmend).

Bei den Geraer Zählungen fehlt die Zeitangabe, doch ergibt sich hier die Procentzahl übereinstimmend höher als für Greiz: z. B. bei H für K_3C_8 46%; bei K (für das die Bemerkungen bei G zutreffen) sogar gegen 70%.

Nach der Frequenz ordnen sich die Hauptformen in den verschiedenen Beobachtungsreihen wie folgt:

Greiz	A	K_3C_8, K_5C_8, K_5C_{10}, K_3C_{10}, K_3C_5, K_6C_8, K_5C_{12}, K_3C_{12}, K_5C_5
	B	K_3C_8, K_5C_8, K_5C_{10}, K_3C_{10}, K_5C_{13}, K_6C_8, K_3C_5, K_3C_{12}, K_5C_5
	C	K_3C_8, K_5C_8, K_5C_{10}, K_3C_5, K_3C_{10}, K_6C_8.
Gera	H	K_3C_8, K_5C_8, K_3C_{10}, K_5C_{10}, K_3C_5, K_5C_{12}, K_3C_{13}
	I	K_3C_8, K_3C_{10}, K_5C_8, K_5C_{10}, K_5C_5, K_3C_5
	K	K_3C_8, K_5C_8, K_5C_{10}, K_3C_{10}.
Trogen		K_3C_8, K_3C_{10}, K_5C_8, K_5C_{10}, K_3C_5.

Es findet sich mithin allenthalben (Greiz, Trogen, Gera) neben K_3C_8 als *Normalform* zunächst mit wechselnder Prävalenz K_5C_8, K_5C_{10}, K_3C_{10}, dann die Formen K_3C_5, K_6C_8, K_5C_{12}, K_3C_{12}, K_5C_5 in noch weiter wechselnder Prävalenz und geringster Frequenz. Die Bedeutung dieser Formen als "petites espèces" liesse sich nicht besser erweisen, als durch die Auffindung einer derselben als Normalform in anderer Gegend. Thatsächlich ist dies geglückt, bevor das vorliegende Material verarbeitet wurde. Herr A. Heyer, Institutslehrer in St. Gallen, erhielt 1900 zunächst 79 Schülerzählungen aus dem 900 m. hoch gelegenen Gais —in Trogen, seinem damaligen Wohnort war *Ficaria verna* bereits verblüht—die die grösste Frequenz für K_5C_8 ergaben. Herr Reallehrer Freund konstatierte sodann, dass dies die Normalform sei. Das Ergebniss war:

Kronblätter

Kelchblätter	6	7	8	9	10	11	12	
3		1	3	5				9
4		4	29	10	2			45
5	1	8	**56**	36	17	6	2	**126**
6		1	1		2			4
	1	14	**89**	51	21	6	2	184

Normalform K_5C_8
Nebenform K_5C_{10}
(und K_3C_8)

Im Jahr 1901 ergaben sich auf demselben Standort noch höhere Zahlen

Kronblätter

Kelchblätter	6	7	8	9	10	11	12	13	14	
2					1	1				2
3		2	25	25	26	10	3			91
4		11	46	98	94	31	3			283
5	2	19	68	140	**172**	132	63	18	2	**616**
6		2		1	3	1				7
7				1						1
	2	34	139	265	**296**	175	69	18	2	1000

also K_5C_{10} als Normalform;
Nebenformen
K_5C_8, K_5C_{10},
K_5C_9, K_5C_{13}.

Dass es sich bei der *Ficaria verna* von Gais um eine andere Rasse als die gewöhnliche handelt, zeigt auch der Vergleich der Korrelationsfelder von Staubgefässen und Stempeln (Zählungen von 1900).

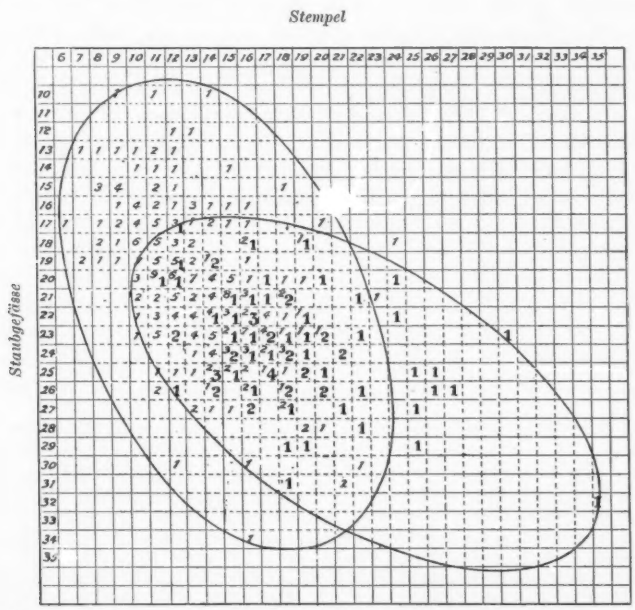

Fig. 1. Felder der korrelativen Variation der Staubgefässe und Stempel der gewöhnlichen Rasse von *Ficaria verna* und der Rasse von Gais (dicke Zahlen).

Zusammengehörige neuere Zählungen scheinen für die bei uns häufigste Form der *Ficaria* $K_3C_8S_{21}G_{13}$ zu ergeben (*S* Staubgefässe, *G* Stempel)*.

II.

Kontinuierliches oder rhythmisch pausierendes Wachsthum?

Wie die Variation in der Anzahl der Blätter etc. eines Stengels, der Blüten eines Blütenkopfes, *sprungweise* erfolgt und zwar nach der Reihe der Fibonaccizahlen und der von mir ermittelten Unterreihe derselben, so dass z. B. bei der Züchtung mehrblütiger Rassen von *Chrysanthemum segetum* (durch de Vries) nicht Rassen mit beliebigen Gipfelzahlen in den Variationscurven, sondern nur solche mit Gipfeln bei 13 16 21 26 34 42 47 etc. Randblüten entstehen konnten, so machen es gewisse Erscheinungen auch wahrscheinlich, dass das Wachsthum (der Stengel, Blätter etc.) gesetzmässig durch gewisse Hauptetappen hindurch läuft, bei denen es enden kann, nicht aber bei beliebigen anderen Werten seinen Abschluss erreicht. Bei den Zellfäden der *Melosira arenaria* (Moore) ist ein solches discontinuierliches rhythmisches Wachsthum allgemein bekannt; die Längen ungleichalosiger Fäden stehen genau in dem Verhältnis der Zahlen der Fibonaccireihe. Auch *bei höheren Pflanzen* deuten die Variationscurven für die Längen etc. der Stengel, Blätter etc. auf ein ähnliches Wachsthumsgesetz hin. So wurde für *Fagus silvatica* etc. nachgewiesen (*Bot. Jaarboek Dodonaea te Gent*, XI. 1899, p. 116) dass das Variationspolygon für die Blattlänge mehrgipfelig ist. Und zwar scheint die Länge des Blattes nicht durch alle Werte hindurch zu variieren, sondern sprungweise und zwar genau nach dem für die numerische Variation der Blütenstande etc. ermittelten Gesetz (cf. *Bot. Centralbl.* 1898 Bd. LXXV. p. 105). Für das Wachsthum der Halminternodien der Getreidearten ergibt sich ähnliches nach den Untersuchungen von de Bruyker (Over correlatieve variatie bij de Rogge en de Gerst, Gent 1898, pp. 42—56; besonders Fig. 4; vgl. meine Bemerkung im *Bot. Centralbl.* Bd. IX. Beihefte 6, 1900 p. 441). *Die bis jetzt vorliegenden Untersuchungen sind aber noch zu geringzählig um daraus weitgehende Schlüsse ziehen zu können. Ein eingehendes Studium von Komplexcurven bei Längenvariation etc. und ihrer*

* Bei der gleichzeitigen Zählung von Kelch-, Blumen-, und Staub-, oder Blumen-, Staub-, und Fruchtblättern bedient man sich zweckentsprechend eines parallelepipedischen Kastens mit parallelen gleicherweise gekästelten Schubfächern in die man Kugeln hineinlegt. Länge, Breite, Höhe entsprechen den 3 korrel. Elementen. Man erhält so einen Korrelationsraum bestimmter, der Species entsprechender Form und Lage—dem zweidimensionalen Korrelationsfeld bei 2 Varianten entsprechend. Eine geometrische Darstellung der Korrelation zwischen allen 4 Blütenteilen ist im dreidimensionalen Raum unmöglich.

Die aus den korrelativen Beziehungen zweier Merkmale abgeleiteten Häufigkeitsbeziehungen zwischen den "petites espèces" lassen sich auch geometrisch anschaulich machen wenn man auf den Quadraten des Korrelationsfeldes senkrechte Stäbe anbringt deren Längen den in ihnen stehenden Zahlen entsprechen, oder den Vertikal- und Horizontalreihen entsprechend zwei senkrecht einander durchkreuzende Systeme der zugehörigen Variationspolygone.

allmählichen Ausgestaltung wird hier allein zum Ziel führen. Ein Anfang wurde auf meine Veranlassung gemacht mit den Nadeln von *Pinus silvestris.* Mein Freund Herr A. Heyer, in St. Gallen, hat von Februar bis April 1900 12000 frische Nadeln einer und derselben ca. 5—6 m. hohen Kiefer bei Trogen abgemessen und zwar, da die Länge der Nadeln von den unteren Ästen zum Gipfel zunimmt, je 4000 von den unteren, den mittleren und oberen Ästen der Krone. Die Gesammtergebnisse zeigt folgende Tabelle:

Pinus silvestris. *Nadellänge in Millimetern.*

Frequenz	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29
	7	8	8	7	17	26	47	66	107	128	160	232	270	307	334	356	336	**362**	307	243	195	**203**	88
			3	12	6	11	7	25	16	21	39	72	70	84	110	127	196	207	**316**	270	290	311	**358**	326
	1	4	12	14	19	22	42	54	55	66	85	84	109	113	142	149	230	**244**	243	274	326	**360**	313
	8	15	32	27	46	55	114	136	183	233	317	386	463	530	603	701	773	**922**	820	807	832	**921**	727

mm.	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49
4000 Nadeln von unteren Ästen	83	45	29	19	4	4	1	1												
4000 Nadeln von mittleren Ästen	276	194	**207**	131	96	83	60	35	26	11	3	1								
4000 Nadeln von oberen Ästen	243	230	154	113	89	53	41	33	26	16	15	7	4	5	1	2	3		2	2
12,000 Nadeln	602	469	390	263	189	140	102	69	52	27	18	8	4	5	1	2	3		2	2

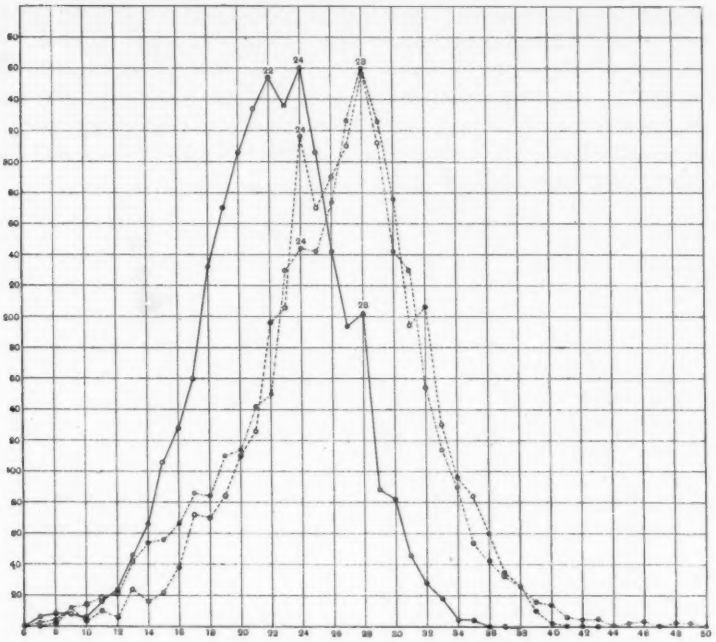

Fig. 2. *Pinus silvestris.* Nadellänge in Millimetern.

(1) ——— Polygon von 4000 Nadeln der unteren Äste.
(2) - - - - - - „ „ „ mittleren „
(3) —•—•—•— „ „ „ oberen „

Vgl. die Polygone in der Abbildung.

Die Hauptcurve erscheint zusammengesetzt aus 2 polymorphen Curven mit den Hauptgipfeln bei 24 und 28 (ersterer teilt die Variationswerte von 7 bis 42 im Verhältnis 2 : 3, letzterer im Verhältnis 3 : 2), sie zeigt einen ganz ähnlichen Bau wie die oben erwähnte Curve von De Bruyker (dessen Fig. 4; nur dass hier die Verhältnisse 5 : 8 und 8 : 5 sind und die Gipfel ungleiche Frequenzen haben), noch mehr erinnern uns die Einzelkurven, die successive zu den obigen Resultaten geführt haben, in ihrem Verlauf an die Fibonaccikurven. [Es ist bemerkenswert dass Heyer schon bei seinen ersten Tausendzählungen immer mehr zu der Überzeugung kam, dass die Gipfel die Multipla einer gewissen "Einheitslänge" darstellten. So zeigte das erste Tausend eine Variation von 7—35 mm., die Gipfellagen bei 21 und 28. Als Einheitslänge betrachtete er 7 mm., $21 = 3 \cdot 7$, $28 = 4 \cdot 7$, $35 = 5 \cdot 7$ wären dann die Multipla—bei den eigentlichen Fibonaccikurven würden nur eben *bestimmte*, nicht *alle* Multipla auftreten wie dies auch in der *grossen Zahl* der Nadelzählungen thatsächlich der Fall ist.]

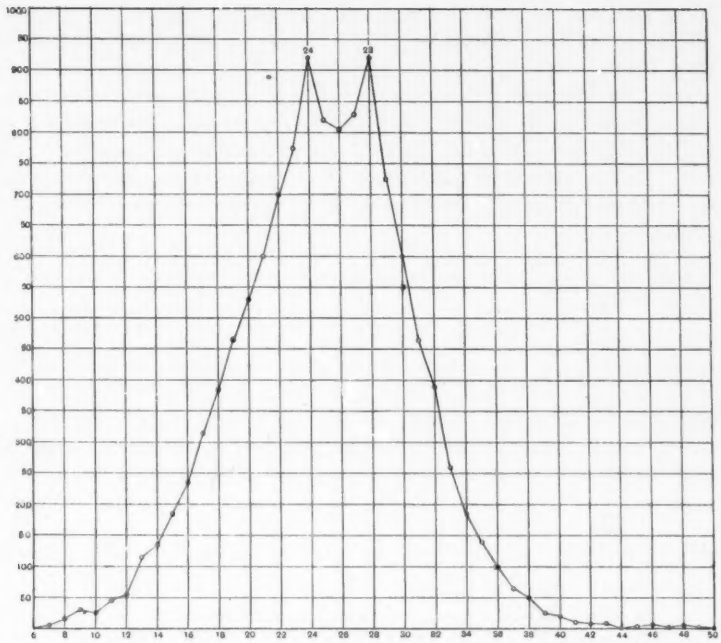

Fig. 3. *Pinus silvestris.* Nadellänge in Millimetern. Summationspolygon der 3 anderen Polygone 12000 Nadelmessungen eines Baumes enthaltend.

Auf die Erreichung *verschiedener, bestimmter Stufen,* bei wenig begrenztem Wachsthum scheinen auch solche Fälle vielgipfeliger Variationscurven bei einheitlicher Art zurückzuführen zu sein, wo die Gipfel von den ersten Zählungen an immer die gleichen bleiben so z. B. bei der Zahl der Ährchen von *Agropyrum repens* (vgl. auch die Blütenstände vieler Papilionaceen, Primulaceen etc.). Hier fand ich bei Schmalkalden und Greiz schon von den ersten Zählungen an überwiegen die Zahlen 11, 13, 17, 19 die auch in dem Gesammtergebnis meiner Zählungen (bei B), und von 348 Zählungen von A. Heyer bei Trogen (A) zu Tage treten:

Zahl der Ährchen bei *Agropyrum repens.*

	6	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	*19*	*20*	*21*	*22*	*23*	*24*	*25*	*26*	*27*
A		3	8	9	18	**28**	22	**35**	29	**32**	21	**22**	18	**32**	25	14	18	8	5			1
B	1	2	8	11	21	**41**	33	**46**	44	37	47	**52**	39	**45**	32	15	10	10	2			
Sa	1	5	16	20	39	**69**	55	**81**	73	69	68	**74**	57	**77**	57	29	28	18	7			1

Diese Zahlen sind auch bei anderen zweizeiligen Ähren mit Gipfelähren ($2n+1$, wo n wie im vorliegenden Fall 5 6 8 9 ist etc.) häufig (bei 4- bezw. 6zeiliger Gerste etc. wird man entsprechend Zahlen der Form $4n+1$, $6n+1$ erwarten). So bei dem polymorphen *Lolium perenne*, dessen Komplexcurve in der einen Gegend (z. B. Trogen 1899, 1900) 11 Ähren, in einer anderen (z. B. Basel 1900) 19 oder (Küsnacht 1900) 17 Ähren aufweist (die einzelnen "petites espèces" dürften aber hier begrenztes Wachsthum haben, da die einzelnen Curven eingipfelig sind); ferner bei dem wenig variablen *Brachypodium pinnatum*:

Zahl der Ährchen bei *Brachypodium pinnatum* (*Trogen, A. Heyer*).

	2	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*
Frequenz	2	5	68	291	584	**709**	697	522	294	172	57	16	3	2

Sa 3422 Expl. (vom Sommer 1900), wo also die 7 ($=2 \cdot 3+1$) überwiegt. Bei dem verwandten *Brachypodium silvaticum* sind die *seitlichen* Ährchen meist unpaarig, denn das (einfache) Variationspolygon wird durch folgende Zahlen bestimmt:

Zahl der Ährchen bei *Brachypodium silvaticum* (*Trogen, A. Heyer* 1900).

	4	*5*	*6*	*7*	*8*	*9*	*10*	*11*	
Frequenz	1	11	81	289	**396**	193	43	6	(1020 Expl.)

III.

Einige weitere Beispiele von Fibonaccicurven.

Korrelation zwischen der Zahl der ♀ Blüten und ☿ Blüten im Blütenköpfchen von *Homogyne alpina*.

Unter den Kompositen liessen sich solche nachweisen bei welchen die Rand- und Scheibenblüten des Körbchens *gleichzeitig* nach höheren bezw. niedereren Zahlen hin variieren—bei ihnen ist der Korrelationscoeffizient positiv—und solche, bei denen beiderlei Blüten *entgegengesetzt* variieren, so dass eine Zunahme der einen Sorte eine Abnahme der anderen und umgekehrt bedingt, bei denen der Korrelationscoeffizient negativ wird. Im letzteren Fall bilden in der Regel erst die Summen der Zahlen der in Korrelation stehenden Glieder Zahlen der Fibonaccireihe etc., im ersteren Falle dagegen schon die Einzelzahlen. Diese Verhältnisse sollen hier nicht weiter erörtert werden, die Zählungen der Zwitterblüten und weiblichen Blüten im Köpfchen bei *Homogyne alpina* müssen noch fortgesetzt werden um den Korrelationscoeffizienten der Species und die

Weibliche Blüten im Köpfchen

	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	
0														1																												1
1																																										
2																																										
3																										1																1
4																	1																									1
5	1																															1										2
6					1			1					1							1						1																5
7												1		1		1	1			2																						7
8												1	1			1	2	1			1	2	2	4							1			1							**21**
9												1			1	1	2	1		1	1	1			1	3			1			2										18
10														1		1	3	2	1	1	1	1	2		1		1	1	1	1	2		1									**22**
11																		1	2		1	1		1							1	1						2			13
12															1	1		1		3	3	3	2	1	1	3	1	1	1	1			1	1				1	1			27
13								1										1			2	1		3	2	3	3	1	3	3	3	1	2	1		1				1		**32**
14															1								1	1	1			1								1						6
15																				1									1						1							3
16																					1																	1				2
17																												1													1
18																																									
19																																									
	1				1			2		1	1	3	2	5	2	6	9	7	3	10	9	10	7	10	5	**12**	9	6	7	5	4	4	4	3	3	3	1	1	2	3	1	162

Zwitterblüten

Variationspolygone für die Zahl der Zwitterblüten bezw. der weiblichen Blüten zuverlässig zu ermitteln. Die vorläufigen hier folgenden Zählungen von Heyer ergaben, dass letztere Fibonaccicurven zugehören und zwar für die weiblichen Blüten mit den Gipfeln bei 13 und 8, für die Zwitterblüten bei 34. (Nach einigen ergänzenden Zählungen von anderen Kollegen in der Schweiz haben die Zwitterblüten wahrscheinlich noch Gipfel bei 21, 29 und 42; die weiblichen bei 10. Die Lage der Korrelationsfelder im Coordinatennetz wird durch diese Zählungen nicht wesentlich geändert.)

Zählungen der Hüllblätter der Köpfchen ergeben gleichfalls Fibonaccicurven mit einem Hauptgipfel bei 13 und Secundärgipfeln bei 16, 10, 21.

Zahl der Hüllblätter	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	*19*	*20*	*21*	*22*	*23*	*24*
Häufigkeit	2	12	**18**	24	61	**153**	57	28	**30**	15	7	7	3	6		3	1

Meine Zählungen im Erzgebirge ergaben für *Homogyne* ähnliche Gipfellage wie für die Schweizer Exemplare.

Arnica montana.

H. de Vries hat gezeigt, dass durch künstliche Züchtwahl auch Rassen von Compositen erzielt werden können, deren Hauptgipfel auf eine der Nebenzahlen der Fibonaccireihe verschoben sind. In der Natur trifft man aber fast stets die Hauptgipfel bei den Hauptzahlen. Eine Ausnahme macht *Arnica montana*, deren Strahlencurve ihren Hauptgipfel bei einer Nebenzahl hat. Verschiedene Zählungen der Strahlenblüten, die ich 1896 am Oberhof in Thüringen, 1900 bei Greiz, Herr Lehrer Auerbach 1900 bei Gera machte, ergaben übereinstimmend den Hauptgipfel bei 16, Nebengipfel bei 13, 18, 21, 10. Das Gesammtresultat war:

Strahlenzahlen	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	*19*	*20*	*21*	*22*	*23*	*24*	*25*	*26*	*27*	*28*	*29*	*30*	*31*	*32*
Frequenz	2	1	**3**	8	15	19	48	76	85	71	**106**	69	73	50	36	27	7	3	2	1	2	1					1

Meine eigenen Zählungen bei Greiz ergaben (vgl. Fig. 4 untere Curve).

7	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	*19*	*20*	*21*	*22*
	2	4	8	8	15	**31**	22	18	**37**	**23**	**29**	**13**	12	**9**	

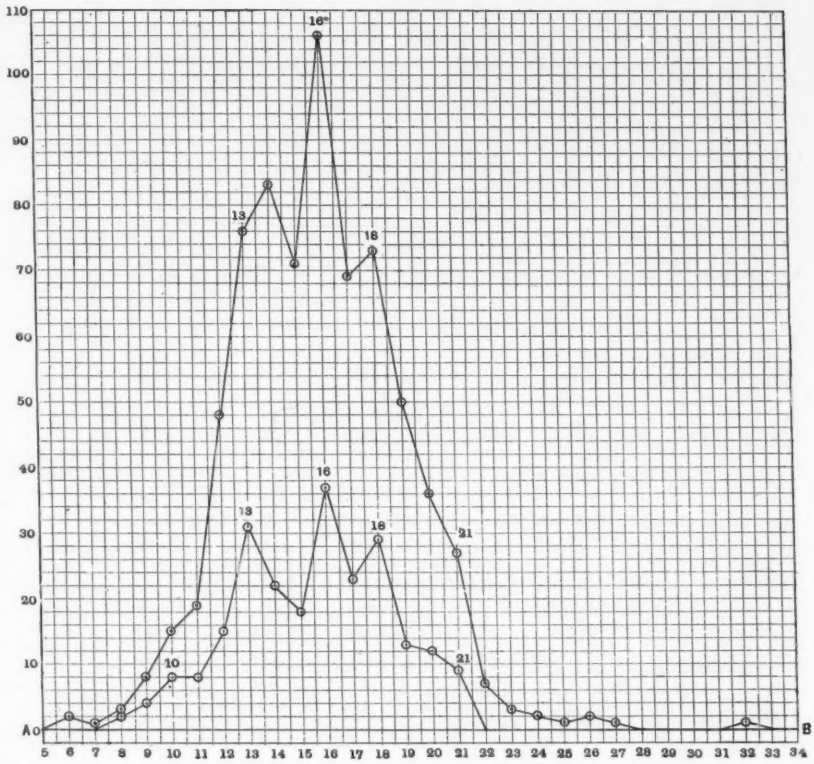

Fig. 4. *Arnica montana* (Strahlencurve).

A. Heyer sandte mir noch 1864 Zählungen von Strahlenblüten des *Bellidiastrum Michelli*, die bei der grossen Variationsweite der Species noch unzureichend zur Gewinnung des specifischen Variationspolygons, doch erkennen lassen, dass dasselbe einer Fibonaccicurve zugehört. Die Curve erstreckt sich von ca. 16 bis 89 und hat allem Anschein nach Gipfel bei 34, 26, 39, 55, 42. *Chrysanthemum Leucanthemum* hat nach seinen Zählungen (340 Exemplaren) eine ähnliche Curve bei Exemplaren aus der Schweiz wie bei den von mir in Deutschland gezahlten Exemplaren.

IV.

Blütendiagramm der Amygdaleen.

In meiner Abhandlung: Über Variationscurven und Variationsflächen der Pflanzen, *Bot. Centralbl.* 1895 Bd. LXIV. habe ich schon auf die Bedeutung der Variationsstatistik für die Ermittelung des Blütenaufbaues im Allgemeinen und der Blütenkreise von Rosaceen (Pomaceen: *Pirus communis*, Amygdaleen: *Crataegus oxyacantha, Crat. coccinea, Prunus spinosa*) im Besonderen hingewiesen. Von besonderem Interesse war der Nachweis des Vorhandenseins *acyclischer neben cyclischer Anordnung* in den Staubgefässkreisen. Das dort erörterte findet weitere Bestätigung durch das Polygon für die Staubgefässzahl von *Prunus spinosa*, wie sie von A. Heyer und mir festgestellt wurde (vgl. auch Fig. 5).

Zahl d. Staubgefässe	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	*19*	*20*	*21*	*22*	*23*	*24*	*25*	*26*	*27*	*28*	*29*	*30*	*31*	*32*	*33*	*34*	*35*
Frequenz	4	1	12	44	56	**77**	63	110	**163**	109	100	98	78	49	49	23	**36**	12	8	9	**14**	1	2	1		1

Die Staubgefässe treten in mehreren Kreisen auf, deren äusserer meist 10 St. enthält, der Innere enthält dann am *häufigsten* 8, (10 + 8 = 18), zuweilen aber nur 3 (10 + 3 = 13), oder 16 (10 + 16 = 26) oder es treten mehrere Kreise (10 + 10 + 10 = 30; 10 + 10 + 10 + 5 = 35) auf. Bei Leipzig fand Dietel nicht 18 sondern 21 als Gipfelzahl (10 + 8 + 3 oder dergl.). Für *Crataegus oxyacantha* ermittelte A. Heyer (Schweiz) übereinstimmend mit mir, ebensowie für *Cr. monogyna* den Gipfel bei 20 (= 10 + 5 + 5).

Zahl der Staubgef.	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	*19*	*20*	*21*	*22*	*23*	*24*	*25*	*26*	*27*	*28*	*29*	*30*	*31*	
Frequenz bei *Cr. oxyacantha*	1		3	2	8	24	46	60	131	174	**242**	102	54	37	12	4	6						908
Frequenz bei *Cr. monogyna*				2	4	21	49	83	174	242	**340**	121	37	12	6	1							1092

Die Einzelkreise für sich würden auch hier reine Fibonaccicurven geben.

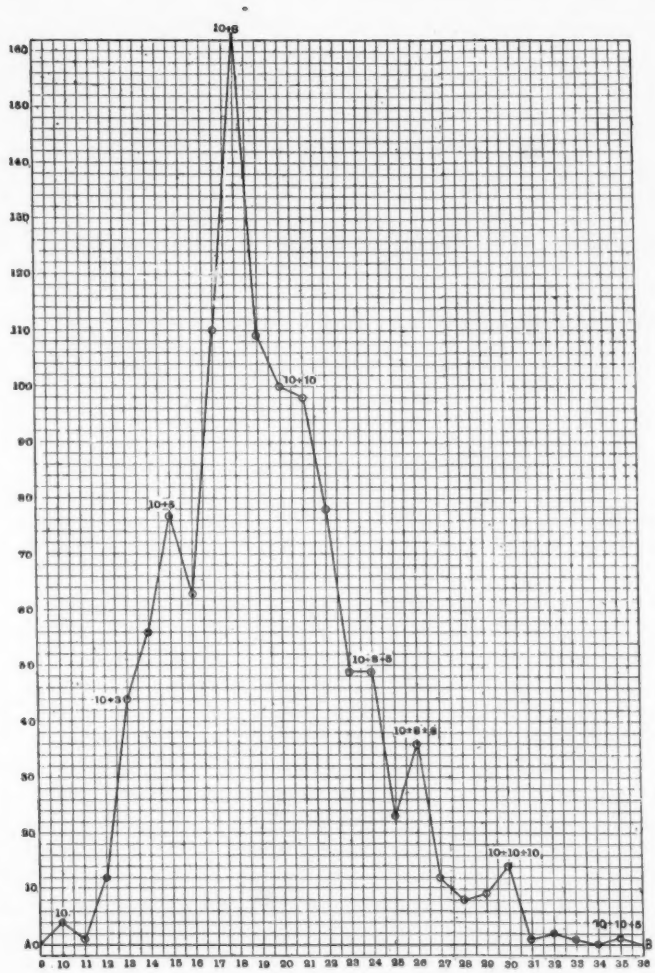

Fig. 5. Zahl der Staubgefässe von *Prunus spinosa*.

DATA FOR THE PROBLEM OF EVOLUTION IN MAN.

ANTHROPOMETRIC DATA FROM AUSTRALIA.

By A. O. POWYS*.

A. *Fertility of Man and Woman in relation to Age.*

To render the problem a definite one the influence of the number of earlier births is practically removed by considering only the *first* births, i.e. those within 12 months of marriage. Table I. gives such *first* births per 100 wives in quinquennial age-groups from the experience of New South Wales during the six years 1893–8. The second column shows the mean annual number of marriages contracted by women in the age-groups indicated in the first column. Körösi, the well-known statistician of Budapest, in a paper read before the Royal Society (*Phil. Trans.*, B, Vol. 186, Part II. pp. 792 *et seq.*), gave a somewhat similar table, in which, however, the probability of birth was shown to be higher the younger the wife. But it must be borne in mind that most of the marriages contracted under the age of 20 are compulsory, i.e. were contracted after conception had followed illicit intercourse. It is therefore only correct that the number of such ante-nuptial conceptions should be withdrawn from both the marriages and the resultant births. The third column shows the number of such ante-nuptial conceptions. Births occurring within 9 months of marriage have been so regarded. It is true that there are some legitimate 7 months children, but the births at that month show no inordinate dimensions, as the following figures indicate. Six years 1893–8: Under 2 months of marriage, 1334 cases; 2—3 months, 1204; 3—4 months, 1577; 4—5 months, 1657; 5—6 months, 1840; 6—7 months, 1942; 7—8 months, 1949; and 8 to 9 months, 1863.

* The following data—tables, calculated constants and diagrams—were freely placed at my disposal by Mr A. O. Powys of the Government Statist's Office, Melbourne. I have added but little to his own notes upon them. K. P.

TABLE I.

Births per 100 *Wives in Age Groups. Experience of New South Wales. Mean Annual Numbers for six years* 1893–8.

Age Group	Mean Annual Marriages	Ante-nuptial Conceptions	Unprejudiced Marriages	Mean Annual Births from 9 to 12 months after Marriage	Births per cent. of unprejudiced Marriages
Under 20	1400	568	832	208	25·0
20 and under 25	3719	1150	2569	995	38·7
25 „ 30	1869	366	1503	596	39·6
30 „ 35	622	98	524	160	30·5
35 „ 40	304	35	269	43	16·0
40 „ 45	133	7·5	125·5	5·2	4·1

The fourth column shows the difference between the second and third columns, leaving what we may term "unprejudiced" marriages. The fifth column shows the births following such unprejudiced marriages within the period 9 to 12 months after marriage, whilst the sixth column gives the percentage ratios between columns four and five. On reference to the diagrams it will be seen that the range extends from 14·5 years to about 45·8 years, with the maximum at about 24·5 years.

The actual curve determined by Mr Powys is

$$y = 40{\cdot}92\left(1+\frac{x}{2{\cdot}0094}\right)^{{\cdot}7746}\left(1-\frac{x}{4{\cdot}2670}\right)^{1{\cdot}6450},$$

with the mean at 27·00 years and mode at 24·49; the unit of x is five years.

As the five year unit of grouping is rather large Dr Alice Lee again worked out the constants of the curve, using Mr W. F. Sheppard's corrections* for the moments. The curve was now found to be

$$y = 41{\cdot}504\left(1+\frac{x}{1{\cdot}6176}\right)^{{\cdot}4893}\left(1-\frac{x}{4{\cdot}1128}\right)^{1{\cdot}2439},$$

with the mean at 27·048 years, the mode at 23·707, and the range from 15·62 to 44·27 years. It may be doubted whether this determination is as good as the previous one, for the curve rises *vertically* at the left-hand terminal of the range, and thus Mr W. F. Sheppard's corrections are shown by the resulting curve to be inapplicable to this case.

Table II. gives the like results for a special tracing from the Register of Victoria of the births resulting from the marriages contracted during the year 1897. On comparing the resulting curve with that for New South Wales it will be seen that the latter is higher in the earlier ages and lower in the later years than the former.

* *Lond. Math. Soc. Proc.*, Vol. xxix., p. 369.

TABLE II.

Births per 100 *Wives in Age Groups. Experience of Victoria from Marriages of* 1897.

Age Group	Mean Annual Marriages	Ante-nuptial Conceptions	Unprejudiced Marriages	Mean Annual Births from 9 to 12 months after Marriage	Births per cent. of unprejudiced Marriages
Under 20	767	338	429	93	21·67
20 and under 25	3122	898	2224	732	32·91
25 „ 30	2235	437	1798	740	41·16
30 „ 35	775	144	631	238	37·72
35 „ 40	349	45	304	60	19·73
40 „ 45	130	16	114	9	7·89

Its equation, the unit of x being five years, is

$$y = 40{\cdot}73\left(1+\frac{x}{2{\cdot}7140}\right)^{1{\cdot}1373}\left(1-\frac{x}{3{\cdot}9254}\right)^{1{\cdot}6449},$$

with the mean at 28·26 years and the mode at 27·00 years. The range is from 13·43 to 46·63 years.

Dr Alice Lee, using Mr Sheppard's corrections, found

$$y = 41{\cdot}06\left(1+\frac{x}{2{\cdot}4113}\right)^{{\cdot}8365}\left(1-\frac{x}{3{\cdot}6969}\right)^{1{\cdot}2826},$$

with the mean at 28·264 years and the mode at 26·703 years. The range is from 14·65 to 45·19 years.

Here again the resulting form of the curve suggests that Sheppard's corrections are unsuitable, and probably the former equation is the better*.

Now whether Mr Powys' or Dr Lee's curves are used we see very clearly:

(*a*) that the fertility of women begins earlier in Victoria and ends later, but

(*b*) that the maximum fertility in New South Wales is reached almost three years earlier than it is in Victoria, say 24 years as compared with 27.

The more rapid development of women in New South Wales is undoubtedly due to its much warmer climate. Nearly one-half of the population of each colony is concentrated in their capital cities; the mean annual temperature of Sydney being 63° F., and that of Melbourne being 57° F.

We should expect the Victorian fertility to end later under these circumstances; that it also begins earlier is noteworthy. The year's advance on New South Wales may not really be a sensible difference; it may result from the large

* Clearly there is not in any of these curves really *high* contact at either end of the range, and the discovery of suitable corrections for the moments in such cases is one we would urge upon any unemployed mathematician. K. P.

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number of ante-nuptial conceptions in the case of the younger brides which have been excluded, or from the difficulty we have referred to about finding the true value of the moments in such large groupings.

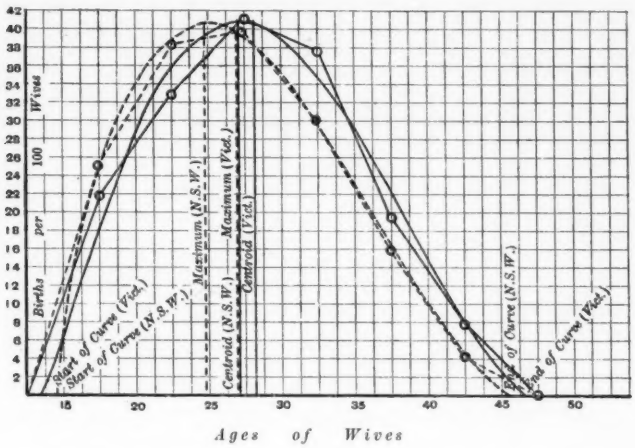

FIG. 1. *Births within 12 months of Marriage (Wives). Victorian Curve, firm lines:*

$$y = 40{\cdot}73\left(1+\frac{x}{2{\cdot}7140}\right)^{1{\cdot}1373}\left(1-\frac{x}{3{\cdot}9254}\right)^{1{\cdot}6449}.$$

Mean: 28·26 years. Mode (origin): 27·00 years. Unit of $x=5$ years.

New South Wales Curve, broken lines:

$$y = 40{\cdot}92\left(1+\frac{x}{2{\cdot}0094}\right)^{{\cdot}7748}\left(1-\frac{x}{4{\cdot}2670}\right)^{1{\cdot}6450}.$$

Mean: 27·00 years. Mode (origin) 24·49 years. Unit of $x=5$ years.

But the problem deserves further consideration. We are accustomed to consider that sexual ripeness is accelerated in warmer climates. Does this mean that the age of *initial* fertility is lowered or that the age of *maximum* fertility is lowered? The one is not unlikely to be confused with the other, if an appeal be made to 'general' observation, and not as in this case to actual statistics.

Table III. gives the like results for first births of the wives in the case of husbands based on the Victorian experience for 1897.

TABLE III.

Births per 100 *Husbands in Age Groups. Experience of Victoria* 1897.

Age Group	Mean Annual Marriages	Ante-nuptial Conceptions	Unprejudiced Marriages	Mean Annual Births from 9 to 12 months after Marriage	Births per cent. of unprejudiced Marriages
Under 20	55	29	26	2	7·70
20 and under 25	1708	591	1117	257	23·00
25 „ 30	2751	702	2049	743	36·25
30 „ 35	1637	334	1303	565	43·35
35 „ 40	717	149	568	191	33·63
40 „ 45	266	43	223	69	30·94
45 „ 50	132	16	116	26	22·41
50 „ 55	95	13	82	10	12·19
55 „ 60	70	0	70	7	10·00
60 „ 65	56	2	54	1	1·85
65 „ 70	33	1	32	1	3·12
70 „ 75	19	1	18	0	0·00

The equation to the curve, the unit of x being five years, is

$$y = 39{\cdot}75\left(1+\frac{x}{3{\cdot}6123}\right)^{1{\cdot}6321}\left(1-\frac{x}{11{\cdot}0331}\right)^{4{\cdot}7857},$$

with the mean at 36·71 years and the mode at 32·41. The range is from 14·34 to 87·57 years.

Unfortunately we have not the New South Wales experience for comparison, but we can hardly doubt that the age of maximum fertility would be somewhat younger.

An examination of Figures 1 and 2 will show how very satisfactorily these curves of Type I.* express the biometric data thus found.

They show quite clearly that at any rate for the Anglo-Saxon race the view of Kőrösi that the fertility is greater the younger the wife is not correct. The woman reaches her greatest reproductive vigour between 24·5 and 27 years, according to the climate, and the man at about 32 years. As we shall see under Section C these appear also to be the ages in the two sexes of maximum physique.

B. *On the Correlation between the Duration of Life and the Number of Offspring.*

The importance of this correlation as a fundamental factor in the quantitative theory of evolution has already been insisted upon†. Another determination of

* *Phil. Trans.*, Vol. 186 A, p. 367.

† Data for the Problem of Evolution in Man, V. On the correlation between the Duration of Life and the Number of Offspring. Beeton, Yule and Pearson, *R. S. Proc.* Vol. LXVII. p. 159.

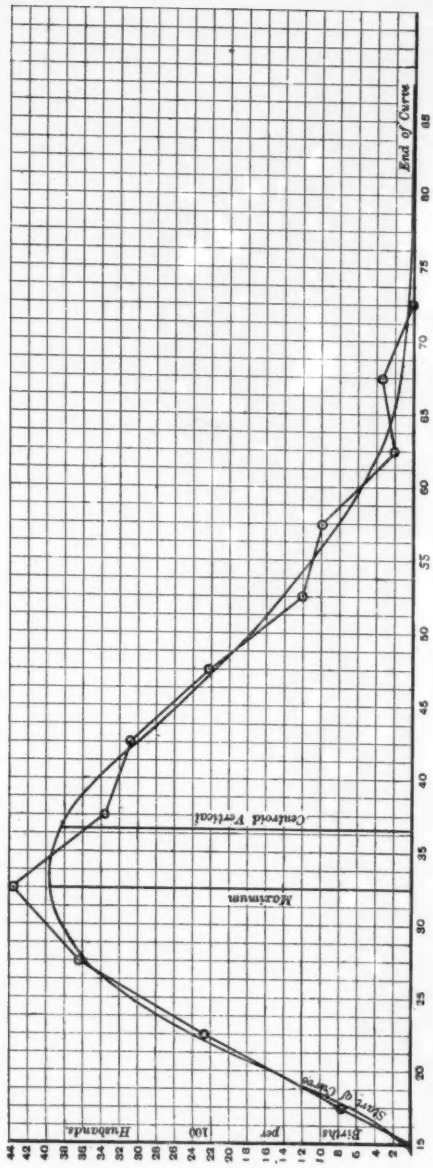

Fig. 2. *Births within 12 months of Marriage (Husbands). Victorian Curve.*

$$y = 39{\cdot}75\left(1+\frac{x}{3{\cdot}6123}\right)^{1{\cdot}6321}\left(1-\frac{x}{11{\cdot}0331}\right)^{4{\cdot}9857}.$$

Mean: 36·71 years. Mode (origin): 32·41 years. Unit of $x = 5$ years.

it has thus considerable interest. Fig. 3 shows the general agreement between New South Wales experience and the statistics for English mothers of Beeton and Pearson*. The Australian statistics are given in Table IV. below.

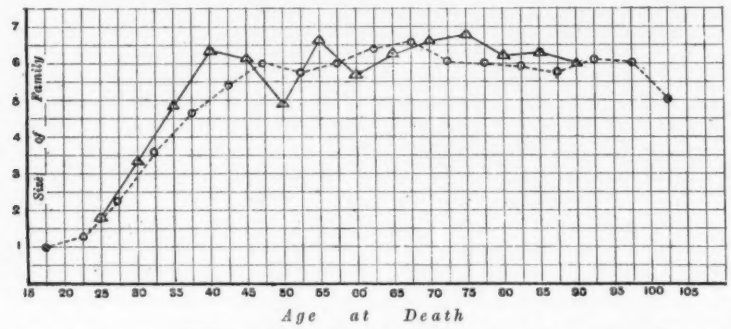

FIG. 3. *Comparison of English and New South Wales Experience as to Number of Offspring and Length of Life of Mother.*

Pearson, firm lines. Powys, broken lines.

TABLE IV.

Ages of Mothers at Death and Number of Children (Living and Dead). Experience of New South Wales 1898 *and* 1899.

Age	Deaths of Mothers			Number of Children			Average number of children to each mother
	1898	1899	Total	1898	1899	Total	
Under 20	28	23	51	26	24	50	0·98
20 and under 25	121	121	242	149	149	298	1·23
25 „ 30	216	175	391	520	384	904	2·31
30 „ 35	259	243	502	915	904	1819	3·62
35 „ 40	259	270	529	1289	1265	2554	4·83
40 „ 45	199	218	417	1094	1189	2283	5·48
45 „ 50	201	200	401	1295	1124	2419	6·03
50 „ 55	192	198	390	1077	1182	2259	5·79
55 „ 60	229	242	471	1315	1536	2851	6·05
60 „ 65	249	258	507	1659	1619	3278	6·47
65 „ 70	268	281	549	1742	1883	3625	6·60
70 „ 75	225	249	474	1339	1516	2855	6·02
75 „ 80	258	243	501	1550	1462	3012	6·01
80 „ 85	172	175	34.	1109	966	2075	5·98
85 „ 90	93	102	195	564	554	1118	5·73
90 „ 95	25	24	49	160	142	302	6·16
95 „ 100	12	10	22	60	72	132	6·00
100 and over	1	1	2	8	2	10	5·00
Totals:	3007	3033	6040	15,871	15,973	31,844	

* *R. S. Proc.* Vol. LXVII. p. 162 and diagram p. 178.

Mr Powys' statistics have been fitted with a cubical parabola in Figure 4*. Its equation is

$$y = 6{\cdot}165{,}263 + 0{\cdot}160{,}401x - 0{\cdot}047{,}676x^2 + 0{\cdot}002{,}788x^3,$$

the origin being at 57·5 years and the unit of x being five years. In fitting a range of 85 years, i.e. 15 to 100 was taken, omitting the data for 100—105 as too few to give a trustworthy average. The corresponding curve for English mothers is

$$y = 6{\cdot}4092 + 0{\cdot}079{,}120x - 0{\cdot}052{,}719x^2 + 0{\cdot}005{,}717x^3$$

with the same origin and unit for x.

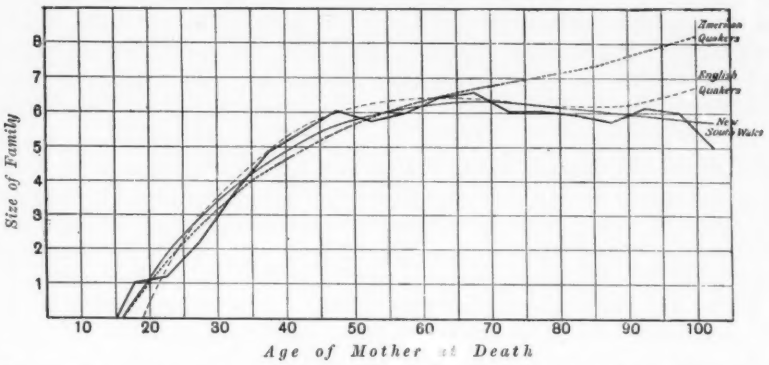

Fig. 4. *Theoretical Curves for Duration of Life and Number of Offspring. Great Britain, America, New South Wales. With New South Wales Experience.*

In Figure 4 the theoretical curves for English, American and Australian mothers are plotted on the New South Wales observations, and the comparison is of considerable interest. The English Quakers, it has been noted, show a tendency to have lessened offspring with parents living to great ages, and this tendency is even more marked in the case of New South Wales. Although the extreme tails, 90—100 years, of all three curves are of small value, yet there certainly appears to be a difference between the American on the one hand and the English and Australian mothers on the other. We are now more justified than we were in June 1900†, in asserting that extreme longevity is not in all cases favourable to a maximum reproductivity. But our warning against rushing to hasty conclusions is confirmed, for a new community like New South Wales with plenty of food and occupation for all is seen to resemble an old community like England in this relationship, and not a new community like America. Under certain conditions we thus see that Weismann's views may be correct: women living to 60—70 may leave more offspring than either those who die younger or those who die older.

* I am responsible alone for this bit of work. K. P.

† *Loc. cit.* p. 166.

In other words, reproductive selection may tend to preserve the threescore years and ten set to the duration of man's life by the Psalmist. But the American statistics show that this tendency is not universal, and further, whether it exists or not, does not seem to depend on the relative density of population and the character of the environment.

C. *On the Stature of Man between the Ages of* 15 *and* 85.

Tables VI. and VII. give the heights of male and female criminals, with their ages based upon New South Wales experience. The criminals from 20 years upwards are for the years 1890–9, and under 20 years for the years 1895–9, each year's record being for *distinct* persons. Mr Powys writes: "I can imagine no solid reason why the heights of criminals should differ in any material way from those of the ordinary population, and the figures shown can therefore be regarded as typical of the latter."

In Australia probably a majority of the population live an outdoor life with plenty of food and exercise, and the differentiation of the criminal from other classes may be small. In Great Britain there is undoubtedly in strength and stature a considerable class differentiation. The upper middle classes are distinctly taller than the labouring classes, and in particular than the town handworkers*. The criminal here is probably of inferior physique, not because he is a criminal, but because in the bulk he is drawn from a differentiated class†. The mean stature of 1077 Cambridge undergraduates is 68″·9, and of 811 persons measured in Mr Galton's South Kensington Laboratory in 1884, 67″·9. The mean stature of the New South Wales male criminals is 67″·3, taking ages 25 to 30 for comparison. In the case of the women we have Cambridge 63″·8, South Kensington 63″·3, and New South Wales not exceeding 62″·8. It seems improbable that the general population of New South Wales is shorter than that of England; hence we shall be justified in believing that to some extent the New South Wales criminals are drawn from a class or classes less fully developed, owing to want of nourishment or to general environment, than the population at large‡. But looking at the curves for the *male* criminals grouped in either 5 or 10 year periods we see that the material is extremely homogeneous. In every case the curve is of Type IV.§, in every case the mode is nearly identical with the mean, and in every case the fit is extremely good. For the males we should have got very good results by simply using the normal curve, and thus we have

* See Pearson: *The Chances of Death*, Vol. I. pp. 310—315.

† The school of Lombroso has jumped without a satisfactory *statistical* investigation at the existence of a definite 'criminal' type. Further light on the matter will be found in Dr W. R. Macdonell's paper on Criminal Anthropometry which will be shortly published in *Biometrika*. K. P.

‡ For 1000 British middle-class from my own measurements (without boots) I find 68″·04 for males and 62″·95 for females, both sensibly taller than the New South Wales criminals. K. P.

§ *Phil. Trans.*, Vol. 186 A, p. 376.

a confirmation of the result several times noted that the frequency of statures in a homogeneous group gives very nearly a normal distribution.

With the females the case is somewhat different; the observed distributions are much more irregular. This is a very common experience in statistics for this sex. It is probably to some extent due to greater difficulty in ascertaining the real age of women, and again to a very likely differentiation in stature between women who have borne and those who have not borne children. But admitting the irregularity, we see again that the mode and the mean are in sensible agreement. The distributions are closely normal. In all cases but one (ages 50 and under 60) the curves diverge from normality in the direction of the curve of Type IV. But in this one case of divergence, the curve, which is of Type II., is within the limits of the probable error of random selection a normal curve. It would seem reasonable to conclude that the stature distribution of these homogeneous groups is nearly normal, but that what divergence there is lies in the direction of curves of Type IV.

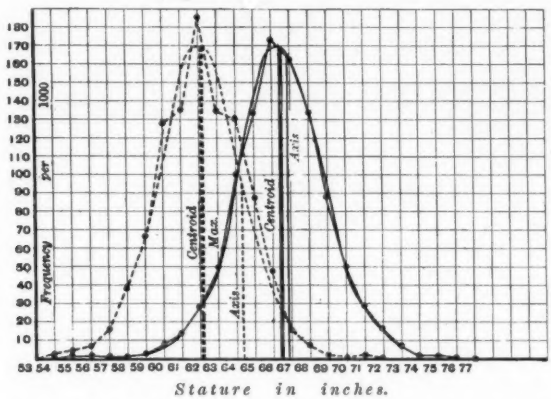

Fig. 5. *Statures per* 1000, *Male and Female, Ages* 20—25. *Male Curve, firm lines:*

$$y = \frac{166{\cdot}7100}{\left\{1+\left(\frac{x}{9{\cdot}9138}\right)^2\right\}^{9{\cdot}5378}} e^{-{\cdot}6617\tan^{-1}\frac{x}{9{\cdot}9138}}.$$

Mean at 66″·95. Origin at 67″·33.
Mode at 66″·99. Modal Frequency = 168·6.
Unit of $x = 1''$.

Female Curve, broken lines:

$$y = \frac{96{\cdot}9970}{\left\{1+\left(\frac{x}{21{\cdot}5568}\right)^2\right\}^{43{\cdot}5885}} e^{-9{\cdot}9124\tan^{-1}\frac{x}{21{\cdot}5568}}.$$

Mean at 62″·60. Origin at 65″·11.
Mode at 62″·66. Modal Frequency = 170·07.
Unit of $x = 1''$.

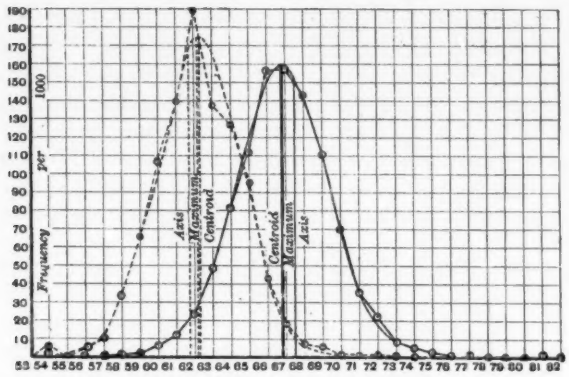
Frequency per 1000
Axis
Maximum
Centroid
Centroid
Maximum
Axis
53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71 72 73 74 75 76 77 78 79 80 81 82
Stature in inches.

FIG. 6. *Statures per* 1000, *Male and Female, Ages* 25—30. *Male Curve, firm lines:*

$$y = \frac{152{\cdot}4356}{\left\{1+\left(\frac{x}{10{\cdot}5901}\right)^2\right\}^{10{\cdot}1018}} e^{-1{\cdot}5405 \tan^{-1}\frac{x}{10{\cdot}5901}}.$$

Mean at 67″·30. Origin at 68″·21.
Mode at 67″·40. Modal Frequency = 161·5.
Unit of $x = 1''$.

Female Curve, broken lines:

$$y = \frac{168{\cdot}71}{\left\{1+\left(\frac{x}{8{\cdot}4961}\right)^2\right\}^{7{\cdot}6360}} e^{{\cdot}9670 \tan^{-1}\frac{x}{8{\cdot}4961}}.$$

Mean at 62″·76. Origin at 62″·14.
Mode at 62″·68. Modal Frequency = 173·8.
Unit of $x = 1''$.

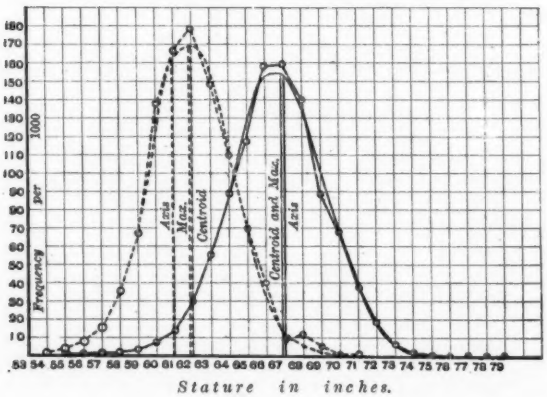
Frequency per 1000
Axis
Max.
Centroid
Centroid and Max.
Axis
53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71 72 73 74 75 76 77 78 79
Stature in inches.

FIG. 7. *Statures per* 1000, *Male and Female, Ages* 30—40. *Male Curve, firm lines:*

$$y=\frac{154{\cdot}8557}{\left\{1+\left(\frac{x}{18{\cdot}6561}\right)^2\right\}^{27{\cdot}5221}}e^{-1{\cdot}0610\tan^{-1}\frac{x}{18{\cdot}6561}}.$$

Mean at 67″·15. Origin at 67″·52.
Mode at 67″·16. Modal Frequency = 156·4.
Unit of $x=1''$.

Female Curve, broken lines:

$$y=\frac{161{\cdot}15}{\left\{1+\left(\frac{x}{10{\cdot}9776}\right)^2\right\}^{12{\cdot}9717}}e^{2{\cdot}3324\tan^{-1}\frac{x}{10{\cdot}9776}}.$$

Mean at 62″·44. Origin at 61″·37.
Mode at 62″·36. Modal Frequency = 178·9.
Unit of $x=1''$.

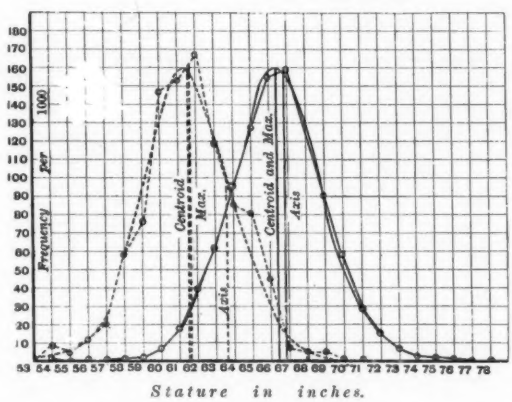

FIG. 8. *Statures per* 1000, *Male and Female, Ages* 40—50. *Male Curve, firm lines:*

$$y=\frac{157{\cdot}20}{\left\{1+\left(\frac{x}{10{\cdot}4170}\right)^2\right\}^{9{\cdot}4312}}e^{-7{\cdot}357\tan^{-1}\frac{x}{10{\cdot}4170}}.$$

Mean at 66″·91. Origin at 67″·37.
Mode at 66″·96. Modal Frequency = 159·16.
Unit of $x=1''$.

Female Curve, broken lines:

$$y=\frac{113{\cdot}67}{\left\{1+\left(\frac{x}{15{\cdot}7508}\right)^2\right\}^{20{\cdot}8386}}e^{-5{\cdot}3107\tan^{-1}\frac{x}{15{\cdot}7508}}.$$

Mean at 62″·12. Origin at 64″·23.
Mode at 62″·22. Modal Frequency = 159·29.
Unit of $x=1''$.

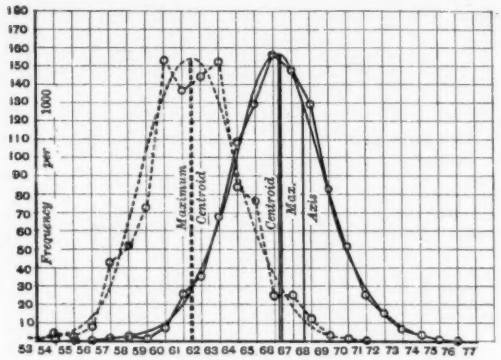

FIG. 9. *Statures per* 1000, *Male and Female, Ages* 50—60. *Male Curve, firm lines:*

$$y = \frac{127{\cdot}74}{\left\{1+\left(\frac{x}{16{\cdot}4710}\right)^2\right\}^{21{\cdot}2579}} e^{-4{\cdot}0223\tan^{-1}\frac{x}{16{\cdot}4710}}.$$

Mean at 66″·74. Origin at 68″·38.
Mode at 66″·62. Modal Frequency = 154·36.
Unit of $x = 1''$.

Female Curve, broken lines:

$$y = 153{\cdot}32\left(1+\frac{x}{19{\cdot}8943}\right)^{38{\cdot}6436}\left(1-\frac{x}{40{\cdot}2965}\right)^{78{\cdot}2314}.$$

Mean at 62″·22. Origin at 62″·05.
Mode at 62″·05. Modal Frequency = 153·32.
Unit of $x = 1''$.

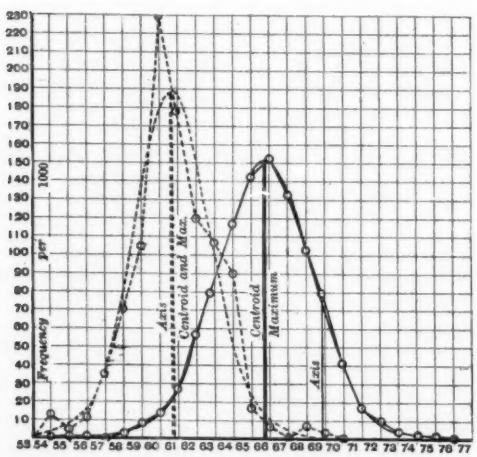

FIG. 10. *Statures per* 1000, *Male and Female, Age* 60 *and over. Male Curve, firm lines:*

$$y = \frac{71{\cdot}00}{\left\{1+\left(\frac{x}{18{\cdot}2028}\right)^2\right\}^{25{\cdot}5423}} e^{-8{\cdot}8304\tan^{-1}\frac{x}{18{\cdot}2028}}.$$

Mean at 66″·26. Origin at 69″·54.
Mode at 66″·39. Modal Frequency = 151·86.
Unit of $x = 1''$.

Female Curve, broken lines:

$$y = \frac{188{\cdot}43}{\left\{1+\left(\frac{x}{6{\cdot}6357}\right)^2\right\}^{5{\cdot}5618}} e^{{\cdot}1179\tan^{-1}\frac{x}{6{\cdot}6357}}.$$

Mean at 61″·31. Origin at 61″·23.
Mode at 61″·30. Modal Frequency = 188·57.
Unit of $x = 1''$.

The following Table gives the means, modes, standard deviations, and skewnesses of the various age groups*.

TABLE V.

Age Group	MEAN		MODE		STANDARD DEVIATION		SKEWNESS	
	Male	Female	Male	Female	Male	Female	Male	Female
20—25	66″·95	62″·60	66″·99	62″·66	2″·4745	2″·3654	+·0163	+·0243
25—30	67″·30	62″·76	67″·40	62″·68	2″·5624	2″·4317	+·0346	−·0333
30—40	67″·15	62″·44	67″·16	62″·36	2″·5866	2″·3027	+·0052	−·0358
40—50	66″·91	62″·12	66″·96	62″·22	2″·6181	2″·5552	+·0184	+·0396
50—60	66″·74	62″·22	66″·82	62″·05	2″·6337	2″·5911	+·0292	−·0172
60 and over	66″·26	61″·31	66″·39	61″·30	2″·6818	2″·3001	+·0478	−·0644

It will be seen that in every group the mean male stature is *less* than the modal stature, while in every female group, except two, the mean stature is *greater* than the modal stature. In the case of these two exceptions the first is the youngest group, and the second, Ages 40—50, contains a very anomalous set of 14 dwarfs. The skewness of the distribution is in all cases small, and a normal curve would be, as we have indicated, a fair fit. The standard deviations are interesting; in the case of the man they increase with age, or it would appear that old men form a more variable group than young men. This is possibly due to the

* I am responsible for some of the calculations involved in the determination of the constants of this Table, and for the introduction of the following table. K. P.

rate of decadence largely varying with the individual. At any rate, whatever be its source, it is very important in its bearing on all attempts to estimate the intensity of selection, even in the case of other animals than man. We must be careful that there is not an age or growth factor at work, counterbalancing a reduction in variability due to selection. The female groups are more irregular in their results than the male; in particular the last group for women over 60 contains only 282 entries. Still the general tendency is the same—increased variability with old age.

Judging by standard deviations man would appear more variable than woman in stature. If we consider all the groups of equal weight and take the mean we have 66″·885 for the mean stature of men, and 62″·242 for women, the standard deviations being respectively 2″·5928 and 2″·4244. Judged by percentage variation we have for the coefficient of variation in man 3·88, and in woman 3·89, results sensibly equal.

The following Table gives relative experience, the agreement in variability is wonderfully close.

Country	Great Britain				New South Wales	
Class	Upper Class *		Middle Class †		Criminal Classes	
Sex	Male	Female	Male	Female	Male	Female
Mean	68″·93	63″·82	67″·90	63″·30	66″·88	62″·24
Standard Deviation ...	2″·51	2″·42	2″·55	2″·40	2″·59	2″·42
Coefficient of Variation	3·6	3·8	3·7	3·8	3·9	3·9

* Cambridge Undergraduates: see Pearson, *The Chances of Death*, Vol. I. p. 313.

† Visitors to International Exhibition in 1884 measured in Mr Galton's South Kensington Laboratory: see Galton, *Natural Inheritance*, p. 200, and Pearson, *loc. cit.* p. 311.

The British measurements practically include nobody over 51 years of age.

TABLE VI.

Heights and Ages of New South Wales Criminals. 7 *years*, 1893—9.
(*Under* 20 *years of age, experience of years* 1895—9.)

Heights	MALES										
	Ages										
	15–16	16–17	17–18	18–19	19–20	20–25	25–30	30–40	40–50	50–60	60 and over
under 4′ 7″	6	4	3	—	—	1	3	2	7	3	
4′ 7″—4′ 8″	1	1	1	—	—	—	3	1	—	—	2
4′ 8″—4′ 9″	6	—	2	—	—	2	1	2	2	—	2
4′ 9″—4′ 10″	5	5	3	2	1	3	2	3	1	2	1
4′ 10″—4′ 11″	15	9	4	3	2	—	9	6	12	7	3
4′ 11″—5′ 0″	13	13	10	3	2	12	25	27	16	8	25
5′ 0″—5′ 1″	19	34	18	14	8	42	70	99	52	35	44
5′ 1″—5′ 2″	13	32	22	14	14	75	164	209	156	117	80
5′ 2″—5′ 3″	21	43	45	51	32	158	323	462	363	157	164
5′ 3″—5′ 4″	20	48	74	81	66	275	633	852	567	301	230
5′ 4″—5′ 5″	10	53	98	113	112	551	1056	1335	867	481	338
5′ 5″—5′ 6″	13	44	88	99	102	725	1427	1793	1154	569	411
5′ 6″—5′ 7″	17	40	98	129	134	939	2001	2398	1391	682	441
5′ 7″—5′ 8″	6	22	52	100	132	882	2019	2406	1431	655	384
5′ 8″—5′ 9″	3	8	41	55	84	734	1826	2103	1153	568	297
5′ 9″—5′ 10″	1	7	18	30	45	482	1410	1351	807	363	228
5′ 10″—5′ 11″		4	5	18	22	281	876	1027	520	232	123
5′ 11″—6′ 0″		2	5	10	22	151	455	568	263	121	49
6′ 0″—6′ 1″		2	5	5	3	82	287	300	142	65	25
6′ 1″—6′ 2″		1	1	2	5	35	105	122	61	30	11
6′ 2″—6′ 3″					1	8	37	33	27	10	2
6′ 3″—6′ 4″					—	3	11	11	8	1	2
6′ 4″—6′ 5″					1	1	—	5	1		
6′ 5″—6′ 6″							2	1	2		
6′ 6″—6′ 7″							1	1			
6′ 7″—6′ 8″											
6′ 8″—6′ 9″							1				
Totals	169	372	593	729	788	5442	12,747	15,117	9003	4407	2862

TABLE VII.

Heights and Ages of New South Wales Criminals. 7 *years,* 1893—9.
(*Under* 20 *years of age, experience of years* 1895—9.)

Heights	FEMALES										
	Ages										
	15–16	16–17	17–18	18–19	19–20	20–25	25–30	30–40	40–50	50–60	60 and over
under 4′ 7″	1	—	1	—	—	2	6	4	14	2	4
4′ 7″—4′ 8″	—	1	—	1	—	4	1	4	9	1	1
4′ 8″—4′ 9″	—	—	—	1	1	6	10	19	23	5	3
4′ 9″—4′ 10″	—	—	2	—	3	19	18	41	35	28	10
4′ 10″—4′ 11″	2	4	5	4	11	47	57	93	99	35	20
4′ 11″—5′ 0″	2	4	3	13	13	85	113	180	130	49	30
5′ 0″—5′ 1″	—	4	4	19	19	163	187	366	250	101	65
5′ 1″—5′ 2″	1	4	8	19	25	173	239	442	259	90	50
5′ 2″—5′ 3″	1	5	18	20	25	235	326	476	282	96	34
5′ 3″—5′ 4″		3	9	16	24	173	236	399	201	101	30
5′ 4″ —5′ 5″		3	2	14	11	165	217	298	143	56	25
5′ 5″—5′ 6″		—	1	5	7	112	165	185	136	51	5
5′ 6″—5′ 7″		—	1	2	2	59	74	102	75	16	2
5′ 7″—5′ 8″		—	3	—	2	18	33	22	14	16	—
5′ 8″—5′ 9″		1	—	—	—	9	12	27	7	9	2
5′ 9″—5′ 10″						3	10	8	7	2	1
5′ 10″—5′ 11″						—	4	3	1	1	
5′ 11″—6′ 0″						1	1				
6′ 0″—6′ 1″							1				
6′ 1″—6′ 2″							1				
Totals	7	29	57	114	143	1274	1711	2669	1685	659	282

D. *On the Alteration of Stature with Old Age.*

This is an exceedingly interesting point, as the influence of age in the adult has to be allowed for in so many anthropometric investigations. Table VIII. gives the criminal experience of New South Wales for the years 1895-9, and the resulting curves are plotted and smoothed in Fig. 11.

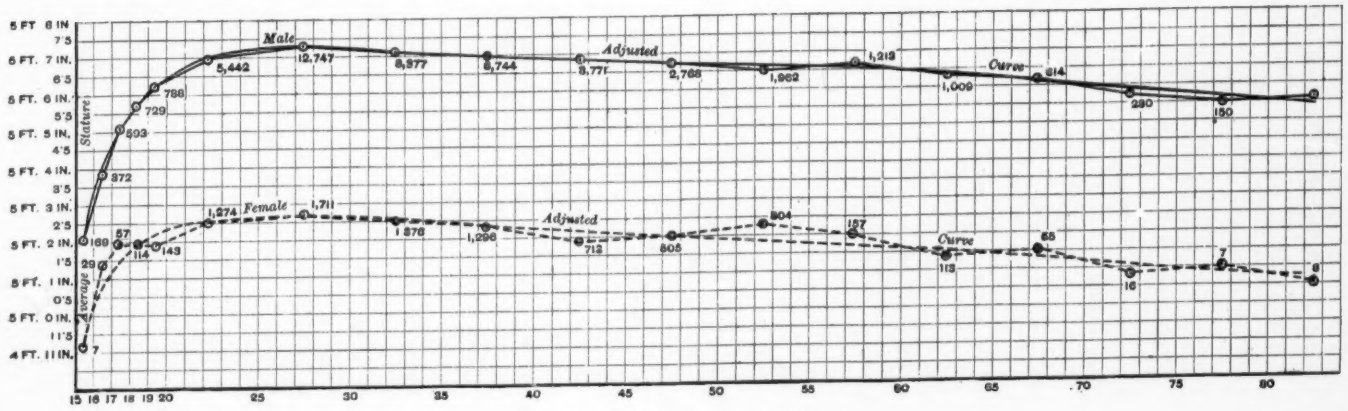

FIG. 11. *Change of Stature with Age.*

TABLE VIII.

Mean Height at various Ages of Males and Females. New South Wales Experience of 5 years, 1895—9.

Ages	Mean Height		Ages	Mean Height	
	Males	Females		Males	Females
15—16	5 ft. 2·12 in.	4 ft. 11·21 in.	50 and under 55	5 ft. 6·58 in.	5 ft. 2·32 in.
16—17	„ 3·83 „	5 ft. 1·40 „	55 „ 60	„ 6·63 „	„ 2·04 „
17—18	„ 5·10 „	„ 2·01 „	60 „ 65	„ 6·40 „	„ 1·31 „
18—19	„ 5·78 „	„ 1·99 „	65 „ 70	„ 6·23 „	„ 1·55 „
19—20	„ 6·37 „	„ 1·93 „	70 „ 75	„ 5·71 „	„ 0·75 „
20 and under 25	„ 6·95 „	„ 2·60 „	75 „ 80	„ 5·49 „	„ 1·07 „
25 „ 30	„ 7·30 „	„ 2·76 „	80 and over	„ 5·67 „	„ 0·50 „
30 „ 35	„ 7·21 „	„ 2·53 „			
35 „ 40	„ 7·07 „	„ 2·33 „			
40 „ 45	„ 6·82 „	„ 1·91 „			
45 „ 50	„ 6·74 „	„ 2·05 „			

The observation curves are remarkably smooth, except in the points for women under 19; but for these youthful criminals there are very few cases. We see at once that the man reaches his maximum stature at about 28 years of age and the woman her maximum stature at about 25. The modal fertility of Victorian women is at 27 and of Victorian men at 32. For New South Wales women the modal fertility is 24·4, two to three years less than for Victoria. We have not the data for New South Wales men, but they would probably show a mode of about 29—30 instead of 32. Thus we see that the age of maximum fertility at any rate approaches, if it does not coincide with, the age of most fully developed stature. As Mr Powys remarks, this tendency of maximum stature age to coincide with that of maximum fertility can hardly be fortuitous. It seems probable that in man, as in other types of life, the age of maximum fertility is the age of most fully developed physique. It will be seen that we have here a definite law of fertility, which is *à priori* reasonable as compared with the result reached by Kőrösi (see p. 34 above)*.

If we examine Mr Powys' diagram, especially the adjusted curves, it would at first sight appear possible to fit them with algebraic expressions. The cubic parabolas determined by Dr Lee are:

* A similar conclusion is formed by M. V. Turquan: Fécondité comparée de l'homme et de la femme suivant l'âge. *Revue scientifique*, T. 5, p. 8, 1896. He makes a maximum fertility for both men and women at the earliest age; he however attributes it to the fact that, "à cet âge la prévoyance malthusienne...est peu connue ou du moins peu pratiquée chez les jeunes mariés de cet âge" (p. 12). His data exhibit, however, a secondary maximum for the father and mother at a later age and there is little doubt that Mr Powys' procedure,—to subtract antenuptial conception—is correct. K. P.

For men:

$$y = 66{\cdot}98419 - {\cdot}056{,}559x - {\cdot}001{,}458x^2 + {\cdot}000{,}0608x^3,$$

the origin being at 50 years of age, the unit of x a year and y being inches of stature.

For women:

$$y = 62{\cdot}17403 - {\cdot}046{,}309x - {\cdot}000{,}095x^3 + {\cdot}000{,}003x^5,$$

the origin being also at 50 years of age, the unit of x a year and y in inches of stature.

But these will be found to give rather poor fits. To see if anything better could be done by taking a higher parabola one of the fourth order was fitted to the male statistics*. Its equation is

$$y = 66{\cdot}521{,}011 - 1{\cdot}979{,}575\left(\frac{x}{35}\right) + 2{\cdot}814{,}332\left(\frac{x}{35}\right)^2 + 2{\cdot}608{,}370\left(\frac{x}{35}\right)^3 - 5{\cdot}352{,}748\left(\frac{x}{35}\right)^4,$$

with origin at 50 years, the unit of x a year and y in inches of stature.

This gave a fairly good fit from 27 to 85 years, because the curve has both points of inflexion in this part, but again fitted badly from 15 to 27 years. In fact from the period of maximum fertility in either sex to the end of life a straight line gives quite as good a fit as it is possible to find. Either the curves selected must therefore be such that they have an asymptote parallel to this line, or we must express by different algebraic expressions the periods of growth and of decadence. In neither case is a parabola of any order whatever a suitable analytical expression. Mr Powys' data being for criminals do not go back sufficiently into childhood to enable us to get a suitable curve for the growth period†.

Starting from the 25—30 group for both sexes we get the following best fitting straight lines for the period of decadence:

$$\text{For males: } y = 68{\cdot}33938 - {\cdot}033{,}6664\,X,$$
$$\text{for females: } y = 63{\cdot}61104 - {\cdot}036{,}1345\,X,$$

where y is the number of inches of stature at X years of age.

It will be found that these lines give excellent results for all ages from 27 to 85. We accordingly conclude that a man loses on the average ·34″ of stature and a woman ·36″ per ten years after 27 and 25 years of age respectively. Approximately we may take this $\frac{1}{3}$″ per ten years. Hence if the stature of a man or woman be measured in adult life we can ascertain by a simple calculation their probable stature in their prime. It is to this stature that we really ought to refer our measurements as a standard: for example, in comparing the stature of parents and children for the purpose of measuring the intensity of inheritance.

* I am responsible for the calculation of the constants of this curve and for those of the straight lines below. K. P.

† I have found fairly good results for the curve of growth of the average child from a logarithmic curve, but my data are at present not sufficient for a good determination. K. P.

ON THE INHERITANCE OF THE DURATION OF LIFE, AND ON THE INTENSITY OF NATURAL SELECTION IN MAN.

By MARY BEETON, Girton College, Cambridge, AND KARL PEARSON.

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(1) IN a first study of the Inheritance of Longevity presented to the Royal Society in June, 1899*, Miss Beeton and I confined our attention to the material then available, namely, inheritance in the male line of direct descent and the correlation between the lengths of lives of brothers. In both cases we dealt only with adult lives. The limitation to the male line was enforced upon us partly by the practice of tracing pedigrees only through the male line, partly by the habitual

* *P. S. Proc.*, Vol. 65, p. 290.

reticence as to the age of women, even at death, observed by the compilers of peerages and family histories. The limitation as to adult lives arose also from the small interest exhibited by the same compilers as to the exact age at death of infants; indeed members of the family dying as infants often escaped any record at all. The difficulties thus referred to were finally overcome by working solely at the pedigree records of members of the Society of Friends. Through the kindness of Mr Isaac Sharpe, a great mass of material in the form of family histories and other records at the central offices of the Society, Devonshire House, E.C., was placed at Miss Beeton's disposal. Further material of a valuable nature was provided by the Secretary of the Friends' Provident Association, and we have cordially to thank both these gentlemen for their ready assistance and their invariable courtesy. Miss Beeton working solely at the material thus provided was enabled to form fairly extensive tables for the inheritance of longevity in the female line, and also with somewhat less extensive data for the deaths of infants* to form some appreciation of what we have elsewhere termed the inheritance of "brachybioty†." The whole of the material was extracted, tabled, and the constants calculated by Miss Beeton‡, and the work has been one demanding very considerable labour and continued caution. My function has been solely consultative in cases where difficulties in dealing with the raw material or in the calculation of the constants arose.

(2) With regard to the tabulation of the data we had found a five year unit of grouping sufficiently close in the case of adult lives. I do not think any serious error is introduced by grouping at the mid-point of five years all the deaths occurring in a five year period. But the matter is very different when we come to deal with individuals dying as minors; we cannot possibly group them together in five year periods, and if we were to do so, then we could not centre the group at the mid-point of the period. The average age at death of children, for example, dying in the first five years of life is certainly not 2·5 years, it is much less. It may be asked why we did not take the year as the unit of life. The answer is twofold: Had we done so we should, treating all lives together, have had to construct a correlation table with over 10,000 compartments! This is practically impossible on account of the labour involved. In the next place the mortality curve is not simple but compound, and the causes of infantile and adult mortality are largely different,—there is, as we have said, a tendency to brachybioty as well as one to longevity. Thus we have separated our tables in the case of those dying as adults and those dying as minors, using a five year period for the classification of the first and a one or a two year period for the classification of the second§. But here again a further difficulty arises. The actual days of birth and

* The whole material available was exhausted, and this must explain why in certain cases we have stopped at less than 1000 cases.

† *R. S. Proc.*, Vol. 65, p. 299.

‡ The whole of the calculations were afterwards corrected or verified by Dr Alice Lee, and we have to thank her most heartily for her aid.

§ Another reason for this division is also to be found in the fact that many infantile deaths are

death are very frequently not stated. Either the individual is said to be n years of age at death, or the years of his birth and death are given. In the former case it would be quite reasonable for adults to assume that the average of such individuals was $n + \cdot 5$ years, for a man n years old at death may be of any age from exactly n to just under $n + 1$ years. In the latter case since the individual may be born any time and die any time between Jan. 1 and Dec. 31, his age may lie between $p - 1$ and $p + 1$ years where m is the given year of his birth and $m + p$ of his death, so that the mere subtraction of the birth and death years gives p years for his age, which corresponds to the probable average life of such individuals if adult. As the years of birth and death are almost always given even when the age is, we obtained the simple rule for classifying: subtract the year of birth from that of death and group on the mid-year of the five year period. Thus ages so found, 21, 22, 23, 24 and 25 are grouped at 23, which denotes exactly 23 years of life and not the mid-point of the 23rd year.

With regard to minors with one and two year periods our difficulties were more serious. p years' difference between the birth and death years still meant p years' life and therefore grouping together lives of p and $p + 1$ years, we could centre the group at $p + \cdot 5$ years, so long as we were not dealing with the first year or two of life and their very unequal mortality. It is these early years of life which present difficulties. If those we have recorded as dying in the same year as they are born, or under the heading 0, were just as likely to be born at any time of the year and to die at any later part of it, their mean life in years would be

$$\int_0^1 \int_x^1 (y - x)\, dx dy \Big/ \int_0^1 \int_x^1 dx dy,$$

where x, y are the periods (in fractions of a year) from January 1 to the days of their birth and death. This gives a result $= \frac{1}{3}$, or the average life would be four months. Mr G. U. Yule kindly considered the problem from the actual statistics provided in the Report of the Registrar-General for 1891. This gives the mortality for each month for the first twelve after birth for the rural counties and three selected towns. Taking the former Mr Yule finds 2·34 months instead of four months for the mean duration of life. As he observes, 2·34 is probably somewhat too large for his material, as undoubtedly more children die towards the beginning than towards the end of the first month. On the other hand the infantile mortality among the Society of Friends is nothing like as great as in the general population. After some consideration we determined to centre the group dying in the first year of life at three months or ·25 years.

Now turning to the group clubbed together as 0—1 in some of our tables, we must ask where it is to be centred. The children recorded as 0 years at death we have seen are to be centred at ·25 years, and those placed under 1 year would be anything from 0 to 2, and accordingly have their mean at 1, if mortality did not

undoubtedly not recorded, and accordingly we have not got infant and adult deaths in their proper proportions. As many infants as possible were extracted, only about 1000 adults.

diminish so rapidly during the first two years of life. I find 917 children on the record under the heading 0, and 774 under the heading 1. Hence we have for the centre of the group $\frac{917 \times \cdot 25 + 774 \times 1}{917 + 774} = \cdot 59$. The real value must be something less than this owing to the rapid diminution of mortality in the first and second years of life. We have accordingly selected ·5 as the centre of the group recorded under 0—1, a value which makes the calculations simpler and cannot be very far from th. ruth. Important as some appreciation of the approximate value of the true centering is, the reader may note that a slight error does not sensibly alter the constants we are investigating. Thus, if we change the centre of the 0—1 group from ·5 to ·64 of a year this, for example in the cases of the correlation in lengths of life of mothers and minor sons, of fathers and infant daughters, only alters the value of the coefficients from ·0756 to ·0761, and from ·0520 to ·0523 respectively. In every case the change is far less than the probable error of the coefficient.

(3) Another point must be referred to here. In forming a correlation table for two perfectly homogeneous groups, e.g. adult sisters and adult sisters, the table was first formed by taking as one entry the elder sister (i.e. the one born first whether she died first or not) and as the other the younger sister. The table was then rendered symmetrical by adding corresponding rows and columns, so that we finally obtained a table giving the correlation between ages at death of a pair of sisters without reference to their relative position in the family. The correlation coefficient was found to be changed by this procedure by an amount less than the probable error. Thus the correlation between the ages at death of pairs of adult sisters in general is $\cdot 3322 \pm \cdot 0185$, while that between elder sisters and younger sisters is $\cdot 3464 \pm \cdot 0183$. In the case of brothers the coefficients are respectively $\cdot 2853 \pm \cdot 0194$ and $\cdot 2990 \pm \cdot 0194$.

But while the correlation remains unchanged by this procedure the means differ very widely. *The elder adult sister and adult brother live on an average four years longer than the younger adult sister or brother.* The actual values based on 1000 or more cases are as follows:

Average Life of Brothers and Sisters.

	All Adult	Elder Adult	Younger Adult
Sister	57·795	59·924	55·667
Brother	56·568	58·560	54·575

Here an elder brother or sister was one who might have been born one year or twenty before the younger brother or sister. This greater expectation of life on the part of the first born was so much beyond our anticipations, and as far as we are aware so little noticed hitherto, that we determined to make an independent investigation of this special point. Accordingly correlation tables were prepared

in which one variable was the number of years' interval between the births of the elder and younger, and the second variable the difference in duration of life of elder and younger. Some remarks must be made on our method of procedure here. The greatest number of years' interval between births was found to be 29. The greatest difference of duration of life 98 years, and this might be either plus or minus: thus if the year was used as unit the correlation table would contain 5684 compartments, and would become very unmanageable. When the table was split into three—adults and adults, minors and minors, adults and minors, the last was still very unwieldy. It was thus necessary to use as unit a five year difference in the duration of life. Using one year difference as unit in the birth interval we found for adults and adults:

	Brothers	Sisters
Mean excess in life of elder ...	4·289 yrs.	4·542 yrs.
Standard deviation of excess...	22·0053 yrs.	22·1325 yrs.
Mean interval between births	6·462 yrs.	6·7503 yrs.
Standard deviation of interval	4·3530 yrs.	4·6856 yrs.
Correlation	·1062 ± ·0206	·1201 ± ·0246
Number of pairs	1051	733

The data are not quite the same as for our pairs of adult brothers and sisters given above, but they show much the same advantage, i.e. four years to the elder. They further demonstrate that longevity is correlated with position in the family. This fact is suggestive for the source of other variations in the characters of an array of brethren. It may be that variability within the array is not purely random but correlated, like variability in longevity, with the birth order. Our numbers show that on the whole the earlier born members of a family are the stronger, or at any rate fitted to survive the longer.

The tables of adults and minors proving very unwieldy we determined to draw up one table only for all brothers and all sisters whatever their age at death, and for this purpose Miss Beeton returned to the Friends' material and extracted the dates of birth and death of 1606 pairs of sisters and 2247 pairs of brothers. These were first classified according to 1, 2, 3 ... years' interval between their births, and then each of these classes subdivided into groups in which the character, life of elder less the life of younger, was nearest to 1, 2, 3 ... 90 or $-1, -2, -3 \ldots -90$ years. Cases in which the difference in duration of life was sensibly zero—very few numerically—were equally divided between the $+1$ and -1 groups. Finally each batch of five years was grouped under its mid-year, and thus Tables I. and II. were obtained. In the column A of these tables is given the number of cases in which for each year of interval between births the younger lives longer; in column B the number of cases in which the elder lives longer. It is only in the case of an interval of one or two years between births that A shows a majority over B. An examination of these columns demonstrates at once the great advantage of priority

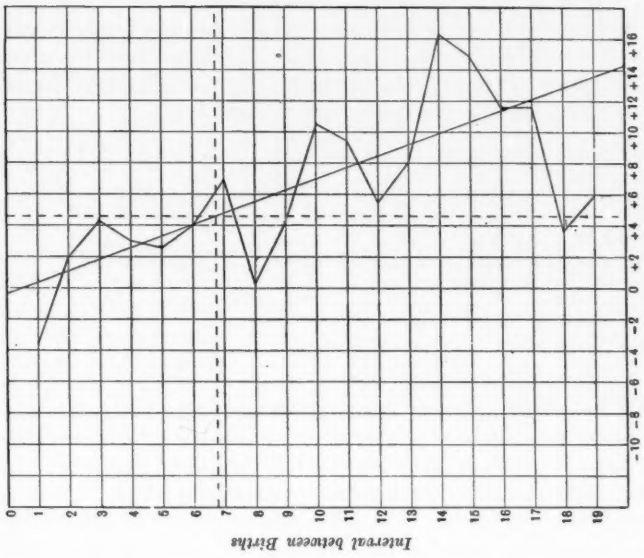

Interval between Births
Mean difference in Duration of Life
(Elder – Younger Brother)

FIG. 2.

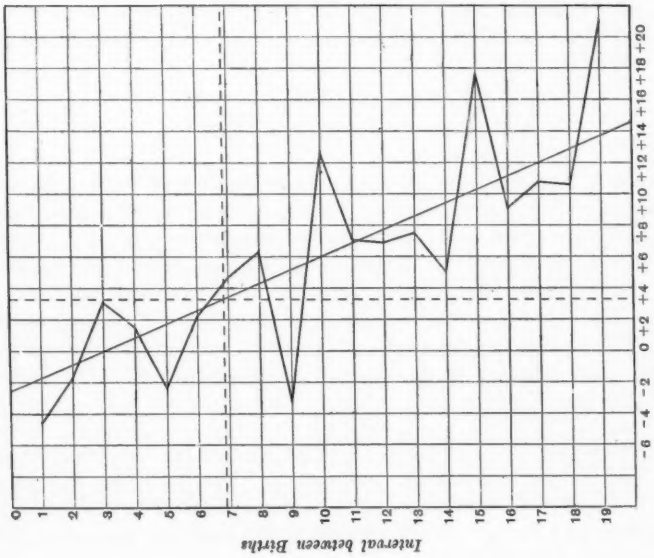

Interval between Births
Mean difference in Duration of Life
(Elder – Younger Sister)

FIG. 1.

of birth as far as longevity is concerned. Out of 1606 pairs of sisters the elder lives longer in 922·5 cases; out of 2247 pairs of brothers the elder lives longer in 1303·5 cases. Or, in 57·4 per cent. of cases the elder sister, in 58·0 per cent. of cases the elder brother lives longer. Summing up our results as for the case of adults we have for all brethren of whatever age at death:

	Brothers	Sisters
Mean excess in life of elder...	4·612 yrs.	3·232 yrs.
Standard deviation of excess...	30·2209 yrs.	29·7849 yrs.
Mean interval between births	6·717 yrs.	6·8400 yrs.
Correlation	·1096 ± ·0141	·1352 ± ·0165
Number of pairs	1606	2247

Now it will be seen that there is no sensible change in the correlation whether we take all pairs of brethren or only adult pairs, nor is there much difference between the mean intervals between births. But the elder brother has slightly gained and the elder sister distinctly lost in excess of life by taking brethren of all ages instead of merely adult pairs. I attribute this to the much larger correlation between the ages at death of infant sisters than we find in the case of infant brothers (see p. 60 below).

The following formulae enable us to predict the probable excess of life (e) of an elder brother or sister from a knowledge of the birth interval (i), both being taken in years:

$$\text{Elder brother: } e = ·7282i - ·2795.$$
$$\text{Elder sister: } e = ·8525i - 2·5991.$$

Thus a brother born ten years before another brother has probably seven years' greater duration of life; a sister born ten years before another sister has probably about six years' greater duration of life. Theoretically the younger brother has the advantage of the elder until $i = ·2795/·7282 = ·38$ years about—an impossible difference. Theoretically the younger sister has the advantage of the elder sister until $i = 2·5991/·8525 = 3·05$ years about, or until the third year say of interval of birth. These results are apparent on Figs. 1 and 2, which give the regression lines, and are quite sufficient in practice for the determination of the probable difference in duration of life.

For the present we have contented ourselves by showing the importance of seniority in birth for the appreciation of longevity. But it will be clear to the reader that the correlations between longevity and actual standing in a family and between longevity and the ages of the parents at the birth are matters of much interest and deserve full investigation.

(4) In an earlier memoir* by one of the present authors some attempt has been made to resolve the mortality curve into components, of which the most important are those of extreme childhood and of old age. The mortality of youth and that of middle age are far less important. In keeping with this resolution we have mentioned in our first study† the existence of an inheritance of brachybioty as well as an inheritance of longevity. Not only for the purposes of manipulating the statistics, but also because we believe there is a real physical distinction, did we consider it desirable to separate brachybioty and longevity. But in order to avoid the immense labour of the cross-correlations if we divided the span of life into four or five sections, and considered how far death in one of those periods influenced duration of life of brethren dying in any one of them, we adopted only a twofold division, distinguishing only between minors and adults. Even this involved the preparation of no less than twelve fraternal correlation tables for duration of life, and in the case of cross-correlations, such as brother or sister minor with brother or sister adult, we were rarely able to collect more than four to five hundred pairs of cases.

Now a noteworthy result of our investigations is this: That while the correlation of adult and adult brethren and of minor and minor brethren are both quite sensible, in no one case is the correlation between the durations of life of an adult and a minor sensible within the limits of our probable error. The following table summarises our results:

Relationship	Correlation	Cases
Adult brother and minor brother	$-\cdot0262 \pm \cdot0246$	753
Adult brother and minor sister	$-\cdot0062 \pm \cdot0349$	374
Adult sister and minor sister	$-\cdot0260 \pm \cdot0291$	537
Adult sister and minor brother	$-\cdot0274 \pm \cdot0328$	421

Now although none of these correlations standing alone are sensible, it is noteworthy that they are all *negative;* an examination of the Tables XIV. XVIII. XXI. and XXII. will at once reveal the source of this. It is the effect of inheritance of the mortality of youth; minors dying from 16 to 20 are associated with adults dying from 21 to 25, i.e. minors dying late correspond to adults dying early. Diagrams of the regression lines‡ show this negative slope in the later years of minority. Here is the source of the negative sign. It might be thought, that if this effect of the mortality of youth were removed, we should get sensible positive correlations between the age at death of an adult brother or sister and of an infant brother or sister. But an examination of the regression lines as far as 10 or 12 years of age shows that there is really but little correlation between the dura-

* *Phil. Trans.*, Vol. 186 A, p. 407.
† *R. S. Proc.*, Vol. 65, p. 299, 1899.
‡ Not reproduced in this paper.

tions of life of adult and infant brethren. Without much more material than we have been able to accumulate I do not believe we could safely break up our data and correlate the duration of life of adults with that of infant brethren dying under 10 or 12 years of age. I think we must conclude: *That to have a brother or sister die as a minor considerably shortens the expectation of life, but whether the brother or sister dies at 2 or 5 or 10 or 15 does not make a difference which we can at present, without further data, consider as sensible.*

The general effect on the duration of life of an adult produced by the death of a brother or sister in their minority is shown in the following table:

Average Age at Death.

If there has died	Of adult brother	Of adult sister
Adult elder brother ...	54·575	57·209
Adult younger brother	58·560	
Minor brother	49·009	46·135
Adult elder sister ...	58·804	55·667
Adult younger sister ...		59·924
Minor sister	48·201	50·300

It is remarkable that the death of his sister as minor seems more important to a man than the death of his brother as minor, and *vice versâ* for a woman. I have dealt with this a little more closely by averaging the age at death of men and women who have lost brothers or sisters in infancy, i.e. before 11 years of age. I find:

Average Age at Death.

If there has died	Of adult brother	Of adult sister
Infant brother	50·116	46·027
Infant sister	47·830	50·236

The probable error is in each case under a year so that the results are significant. One would like to see the whole matter investigated with the ample material which could only be provided by large assurance offices. But our results suggest that some diseases to which a family may be prone are more liable to attack one sex in infancy and the other in the adult period.

The above averages suffice also to indicate that whether a person's brother or sister dies as an infant or between 10 and 21, there is no very large difference in

that person's expectation of life. In the case of the man, indeed, it is an advantage for his brother to die as an infant rather than as an adolescent; this is, of course, due to the effect of the mortality of youth being sensible in the early years of adult life, and influencing the heritage. It appears to have much less influence in the case of sisters. One word here about the influence of environment. There may be some readers who will be inclined to consider that much of the correlation of duration of life between brethren is due to there being a likeness of their environment, and that thus each pair of brethren is linked together and differentiated from the general population. But it is difficult to believe that this really affects *adult* brothers or a father and his adult offspring. A man who dies between 40 and 80 can hardly be said to have an environment more like that of his brother or father, who died also at some such age, than like any other member of the general population. Of course two brothers have usually a like environment in infancy, and their ages at death, even if they die adults, may be influenced by their rearing. But if this be true, we ought to find a high correlation in ages at death of brethren who die as minors. As a matter of fact this correlation for minor and minor is 40 to 50 per cent. less than in the case of adult and adult. It would thus seem that identity of environment is not the principal factor in the correlation between ages of death, for this correlation is far less in youth than in old age.

(5) The following Table summarises our chief results:

TABLE A.

Synopsis of Numerical Results for Inheritance of Duration of Life.

See Table	Relatives		Mean Durations in years		Standard Deviations in years		Correlations	Regressions		No. of Pairs
	x	y	m_x	m_y	σ_x	σ_y	r_{xy}	R_{xy}	R_{yx}	n
(i) DIRECT INHERITANCE.										
III	Father	Adult son ...	68·370 ± ·313	53·490 ± ·417	14·6974 ± ·2217	19·5706 ± ·2952	·1353 ± ·0209	·1016	·1802	1000
IV	Father	Minor son ...	63·847 ± ·333	5·541 ± ·140	15·1829 ± ·2358	6·3758 ± ·0990	·0865 ± ·0218	·2059	·0363	943
V	Father	Adult daughter	69·547 ± ·271	52·535 ± ·429	13·6649 ± ·1917	21·6121 ± ·3032	·1301 ± ·0195	·0822	·2057	1156
VI	Father	Minor daughter	63·573 ± ·366	6·289 ± ·157	15·8439 ± ·2587	6·8212 ± ·1114	·0520 ± ·0230	·1208	·0224	853
VII	Mother	Adult son ...	67·947 ± ·326	54·365 ± ·369	16·9033 ± ·2308	19·1254 ± ·2612	·1313 ± ·0190	·1161	·1486	1220
VIII	Mother	Minor son ...	58·597 ± ·434	5·046 ± ·151	18·1448 ± ·3069	6·3149 ± ·1068	·0756 ± ·0238	·2174	·0263	795
IX	Mother	Adult daughter	68·702 ± ·339	53·011 ± ·435	16·4167 ± ·2400	21·0441 ± ·3077	·1493 ± ·0202	·1166	·1914	1064
X	Mother	Minor daughter	57·656 ± ·446	5·763 ± ·159	18·4962 ± ·3155	6·5852 ± ·1122	·1377 ± ·0237	·3867	·0490	782
(ii) COLLATERAL INHERITANCE.										
XI	Elder adult brother ...	Younger adult brother ...	58·560 ± ·412	54·575 ± ·412	19·3452 ± ·2918	19·3105 ± ·2918	·2290 ± ·0194	·2995	·2985	1000
XII	Adult brother	Adult brother	56·568 ± ·414	56·568 ± ·414	19·4302 ± ·2930	19·4302 ± ·2930	·2853 ± ·0196	·2853	·2853	1000
XIII	Minor brother	Minor brother	4·633 ± ·177	4·633 ± ·177	5·9695 ± ·1252	5·9695 ± ·1252	·1026 ± ·0294	·1026	·1026	517
XIV	Adult brother	Minor brother	49·009 ± ·456	5·532 ± ·159	18·5526 ± ·3225	6·4591 ± ·1123	− ·0262 ± ·0246	− ·0753	− ·0091	753
XV	Elder adult sister ...	Younger adult sister ...	59·924 ± ·424	55·667 ± ·443	20·3812 ± ·3000	21·2689 ± ·3130	·3464 ± ·0183	·3319	·3615	1050
XVI	Adult sister ...	Adult sister ...	57·795 ± ·436	57·795 ± ·436	20·9382 ± ·3082	20·9382 ± ·3082	·3322 ± ·0185	·3322	·3322	1050
XVII	Minor sister ...	Minor sister ...	4·908 ± ·186	4·908 ± ·186	5·8841 ± ·1317	5·8841 ± ·1317	·1748 ± ·0307	·1748	·1748	454
XVIII	Adult sister ...	Minor sister ...	50·300 ± ·602	5·740 ± ·187	20·6847 ± ·4257	6·4123 ± ·1320	− ·0260 ± ·0291	− ·0838	− ·0081	537
XIX	Adult brother	Adult sister ...	58·804 ± ·290	57·209 ± ·315	18·9472 ± ·2048	20·5869 ± ·2225	·2319 ± ·0145	·2134	·2520	1947
XX	Minor brother	Minor sister ...	4·580 ± ·149	4·528 ± ·137	5·8044 ± ·1053	5·3459 ± ·0970	·1435 ± ·0251	·1558	·1322	691
XXI	Adult brother	Minor sister ...	48·201 ± ·682	5·384 ± ·214	19·5545 ± ·4822	6·1350 ± ·1513	− ·0062 ± ·0349	− ·0196	− ·0019	374
XXII	Adult sister ...	Minor brother	46·135 ± ·673	4·857 ± ·202	20·4876 ± ·4762	6·1342 ± ·1426	− ·0274 ± ·0238	− ·0916	− ·0082	421

From this table the following equations, giving the probable age at death of male and female individuals when the ages at death of certain of the relatives are known, have been determined.

Prediction of Age at Death.

Let M = probable age at death of a man.
W = " " " " woman.
F = known age at death of father.
M = " " " " mother.
S_0 = " " " " son.
D = " " " " daughter.
S_0', D' = known ages at death of other sons and daughters.
B, B' = " " " brothers.
S_i, S_i' = " " " sisters.

All the above capital letters mark individuals who live to be 21 or more.

s_0 = known age at death of minor son.
d = " " " minor daughters.
s_0', d = known ages at death of other minors, sons and daughters.
m = probable age at death of a male minor.
w = " " " " female minor.
b, b' = known ages at death of minor brothers.
s_i, s_i' = " " " minor sisters.

All the above small letters mark individuals who die at or before 21 years.

Σ = standard deviation of the array of individuals having the probable age at death given by the equation.

TABLE B.

(1)	$M = 41{\cdot}170 + {\cdot}1802\,F$	$\Sigma = 19{\cdot}3906$
(2)	$M = 44{\cdot}268 + {\cdot}1486\,M$	$\Sigma = 18{\cdot}9598$
(3)	$M = 31{\cdot}500 + {\cdot}1802\,F + {\cdot}1486\,M$	$\Sigma = 19{\cdot}0009$
(4)	$W = 38{\cdot}229 + {\cdot}2057\,F$	$\Sigma = 21{\cdot}4284$
(5)	$W = 39{\cdot}861 + {\cdot}1914\,M$	$\Sigma = 20{\cdot}8074$
(6)	$W = 25{\cdot}317 + {\cdot}2057\,F + {\cdot}1914\,M$	$\Sigma = 20{\cdot}9056$
(7)	$M = 62{\cdot}935 + {\cdot}1016\,S_0$	$\Sigma = 14{\cdot}5622$
(8)	$M = 65{\cdot}229 + {\cdot}0822\,D$	$\Sigma = 13{\cdot}5487$
(9)	$M = 59{\cdot}427 + {\cdot}0790\,(S_0 + S_0')$	$\Sigma = 13{\cdot}8845$
(10)	$M = 61{\cdot}058 + {\cdot}0805\,S_0 + {\cdot}0684\,D$	$\Sigma = 13{\cdot}5959$
(11)	$M = 62{\cdot}415 + {\cdot}0617\,(D + D_0')$	$\Sigma = 12{\cdot}6697$
(12)	$M = 62{\cdot}706 + {\cdot}2059\,s_0$	$\Sigma = 15{\cdot}1260$
(13)	$M = 62{\cdot}813 + {\cdot}1208\,d$	$\Sigma = 15{\cdot}8225$
(14)	$M = 62{\cdot}117 + {\cdot}1867\,(s_0 + s_0')$	$\Sigma = 14{\cdot}9999$

(15)	$M = 62{\cdot}191 + {\cdot}2156\ s_0 + {\cdot}1174\ d$	$\Sigma = 15{\cdot}2830$
(16)	$M = 62{\cdot}564 + {\cdot}1028\ (d + d')$	$\Sigma = 15{\cdot}5640$
(17)	$W = 61{\cdot}635 + {\cdot}1161\ S_0$	$\Sigma = 16{\cdot}7569$
(18)	$W = 62{\cdot}521 + {\cdot}1166\ D$	$\Sigma = 16{\cdot}2320$
(19)	$W = 57{\cdot}728 + {\cdot}0903\ (S_0 + S_0')$	$\Sigma = 15{\cdot}9819$
(20)	$W = 57{\cdot}222 + {\cdot}0899\ S_0 + {\cdot}1017\ D$	$\Sigma = 15{\cdot}9425$
(21)	$W = 58{\cdot}585 + {\cdot}0875\ (D + D')$	$\Sigma = 15{\cdot}2221$
(22)	$W = 57{\cdot}500 + {\cdot}2174\ s_0$	$\Sigma = 18{\cdot}0929$
(23)	$W = 55{\cdot}427 + {\cdot}3867\ d$	$\Sigma = 18{\cdot}3199$
(24)	$W = 56{\cdot}770 + {\cdot}1972\ (s_0 + s_0')$	$\Sigma = 17{\cdot}9552$
(25)	$W = 55{\cdot}290 + {\cdot}1802\ s_0 + {\cdot}4440\ d$	$\Sigma = 17{\cdot}9290$
(26)	$W = 54{\cdot}807 + {\cdot}2903\ (d + d')$	$\Sigma = 17{\cdot}9169$
(27)	$M = 40{\cdot}429 + {\cdot}2853\ B$	$\Sigma = 18{\cdot}6227$
(28)	$M = 46{\cdot}596 + {\cdot}2134\ S_i$	$\Sigma = 18{\cdot}4307$
(29)	$M = 34{\cdot}233 + {\cdot}2447\ B + {\cdot}1633\ S_i$	$\Sigma = 17{\cdot}6049$
(30)	$M = 31{\cdot}455 + {\cdot}2220\ (B + B')$	$\Sigma = 17{\cdot}4034$
(31)	$M = 40{\cdot}476 + {\cdot}1602\ (S_i + S_i')$	$\Sigma = 17{\cdot}1345$
(32)	$W = 42{\cdot}390 + {\cdot}2520\ B$	$\Sigma = 20{\cdot}0257$
(33)	$W = 39{\cdot}259 + {\cdot}3222\ S_i$	$\Sigma = 19{\cdot}7491$
(34)	$W = 30{\cdot}037 + {\cdot}1793\ B + {\cdot}2784\ S_i$	$\Sigma = 18{\cdot}7762$
(35)	$W = 35{\cdot}027 + {\cdot}1961\ (B + B')$	$\Sigma = 18{\cdot}8877$
(36)	$W = 28{\cdot}971 + {\cdot}2494\ (S_i + S_i')$	$\Sigma = 18{\cdot}0391$
(37)	$m = 4{\cdot}158 + {\cdot}1026\ b$	$\Sigma = 5{\cdot}9380$
(38)	$m = 3{\cdot}875 + {\cdot}1558\ s_i$	$\Sigma = 5{\cdot}7443$
(39)	$m = 3{\cdot}565 + {\cdot}0837\ b + {\cdot}1448\ s_i$	$\Sigma = 5{\cdot}7498$
(40)	$m = 3{\cdot}771 + {\cdot}0931\ (b + b')$	$\Sigma = 5{\cdot}8843$
(41)	$m = 3{\cdot}278 + {\cdot}1326\ (s_i + s_i')$	$\Sigma = 5{\cdot}6140$
(42)	$w = 3{\cdot}923 + {\cdot}1322\ b$	$\Sigma = 5{\cdot}2906$
(43)	$w = 4{\cdot}050 + {\cdot}1748\ s_i$	$\Sigma = 5{\cdot}7935$
(44)	$w = 3{\cdot}442 + {\cdot}1163\ b + 1547\ s_i$	$\Sigma = 5{\cdot}4308$
(45)	$w = 3{\cdot}417 + {\cdot}1199\ (b + b')$	$\Sigma = 5{\cdot}2214$
(46)	$w = 3{\cdot}447 + {\cdot}1488\ (s_i + s_i')$	$\Sigma = 5{\cdot}6408$
(47)	$m = 3{\cdot}223 + {\cdot}0363\ F$	$\Sigma = 6{\cdot}3519$
(48)	$m = 3{\cdot}505 + {\cdot}0263\ M$	$\Sigma = 6{\cdot}2968$
(49)	$m = 1{\cdot}434 + {\cdot}0363\ F + {\cdot}0263\ M$	$\Sigma = 6{\cdot}3033$
(50)	$w = 4{\cdot}865 + {\cdot}0224\ F$	$\Sigma = 6{\cdot}8120$
(51)	$w = 2{\cdot}938 + {\cdot}0490\ M$	$\Sigma = 6{\cdot}5224$
(52)	$w = 1{\cdot}777 + {\cdot}0224\ F + {\cdot}0490\ M$	$\Sigma = 6{\cdot}6307$

Thus equation (2), for example, gives the probable age at death of a man when the ages at death of his father and mother are known. These equations have been determined by the usual regression formulae of the theory of correlation; but certain assumptions have had to be made in determining them. Thus from Table A we see that the variability for their age at death of adult sons of fathers

is expressed by $\sigma_y = 19{\cdot}5706$ years, and of adult sons of mothers by $\sigma_y = 19{\cdot}1254$ years. Now to obtain our data we had to take such material as was available and we could not select only those sons whose parents' ages at death were *both* recorded. Accordingly our series of sons is not the same in the case of mothers as it is in that of fathers, and we have taken in our regression equation the mean σ_y as determined from both series. Again, in judging of the probable age at death of a man from those of his brother and sister, we find that the variability of age at death of a man with an adult brother is given by $\sigma_x = 19{\cdot}4302$ years, but of a man with an adult sister as 18·9472 years. Further, the mean age at death of a man with an adult brother is 56·568 years, but with an adult sister is 58·804 years. These are sensible differences, and there can hardly be a doubt that men with sisters live slightly but sensibly longer than those with brothers, and are slightly but sensibly less variable in their age at death. Now our data do not provide the constants for the specially differentiated class of men with *both* brothers and sisters. Accordingly we have been compelled to take the mean of the two classes —men with brothers and men with sisters—to represent both in age at death and variability the special class of men with both a brother and a sister. Other similar cases will occur to the reader, and there are some in which the assumptions made are less justifiable than the above. For example take the case of a minor having two minor brothers dying. We have only been able to use the mean age at death of a minor having at least one minor brother dying, but this is certainly greater than the mean age at death of a minor having at least two minor brothers dying. We were only able to obtain altogether 517 cases of minor brothers dying out of all the records of the Society of Friends accessible to us. It would have been idle to have attempted the differentiation of this small number into sub-classes of minors with one or with more than one brother dying as minor, or with one brother and with one sister dying as minor, and so on. Our equations do not pretend to give more than a rough appreciation, such as is compatible with the comparative paucity of our material, of the influence of the death of relatives on the probable age at death of any individual. Just because we consider the record of the deaths of minors to be very incomplete even in the case of Friends' family histories, while that of adults is fairly complete, we have purposely avoided the important problem which lies at the root of much of the practical use of equations (37) to (52), namely: What are the chances *in the material we are working on* that an individual will die as a minor or survive to be an adult? These chances can be determined for the general population from the Registrar-General's returns, but our material is considerably differentiated from the general population and we have felt bound to leave this problem unanswered.

(6) Before we proceed to illustrate these equations for the probable ages at death, it may be as well to compare, as far as is possible, our present data with what we gave in our first study of the inheritance of longevity*. We were then working solely with male inheritance and from different classes, those of the

* *R. S. Proc.*, Vol. 65, pp. 297, 299.

landed gentry and of the peerage. We have the following comparative results:

Constant	(1) Peerage	(2) Landed Gentry	Mean of (1) and (2)	Friends
M_F	65·835	65·963	65·899	68·370
M_S	58·775	60·915	59·845	53·490
M_B	60·971	—	60·971	56·568
σ_F	14·6382	14·4308	14·5345	14·6974
σ_S	17·0872	17·0986	17·0927	19·5706
σ_B	16·8354	—	16·8354	19·4302
r_{SF}	·1149	·1418	·1283	·1353
r_{BB}	·2602	—	·2602	·2853

Here M_F, M_S, M_B are the mean ages at death of fathers, sons, and brothers; σ_F, σ_S and σ_B are the standard deviations of the same classes respectively, and r_{SF}, r_{BB} are the correlations of ages at death of father and son and of pairs of brothers.

Now these classes are really widely differentiated. Many of the Friends belong to the farming and small trading, as well as to the greater commercial classes. We see at once that the fathers live longer and the sons live shorter lives than is the case with the peerage or landed gentry. The younger male Friends have also a greater variety in their ages at death. Yet notwithstanding these class differences the values of r_{SF} and r_{BB} are in substantially good agreement for the two investigations. The value of r_{SF} for the Friends lies between those for the peerage and the landed gentry; the value of r_{BB} differs from that for the peerage by less than the probable error of the difference.

If we compare the six diagrams of plotted regression polygons and their best fitting regression lines which accompany this paper with the three diagrams of the former paper we shall see that the two sets are in general agreement. We note that the straight line serves as well as any curve could do to give the general drift of the phenomena; the chief deviations from it are as a rule at early or late ages, where the data themselves are very sparse. We find also the same sensible divergence from the theoretical values for parental and fraternal inheritance marking the existence of a considerable non-selective death-rate. Fig. 3 for the regression of fathers on sons even suggests the same points as Fig. (i) of the earlier memoir*. In early manhood the inheritance of duration of life approaches much more nearly the slope of the theoretical line; in middle life the correlation is small, the non-selective death-rate having greater influence; while in old age the inheritance of longevity is again very manifest†.

With standard deviations for father and son of 14·6974 years and 19·5706 years, and a theoretical correlation of ·3 according to the Galtonian law, the slope

* *Loc. cit.* p. 296. † *Loc. cit.* p. 299.

of the regression line of father or son ought to be ·2253; for adult sons it is only ·1016, but an examination of the diagram shows that it is much nearer ·2 than ·1 for early manhood. Further, our own results show that it is actually ·2059 for a father and a minor son. Thus our original statement* that in youth and infancy the regression line for parent and child would approach nearer its theoretical position seems to receive confirmation. The tendencies which cause an individual to have a short life, i.e. to die before the age of 35, are largely inherited tendencies, and they shorten also the life of the parent. The tendencies which lengthen the life of an individual beyond sixty years are also inherited, and they appear as lengthening the parental life. But between 35 and 60 the relationship of duration of life in parent and offspring is not very marked. We thus can distinguish an inheritance of brachybiotγ separated from an inheritance of longevity by a period of life in which the non-selective death-rate is predominant. The whole of

FIG. 3.

Regression Line: Fathers on Sons.

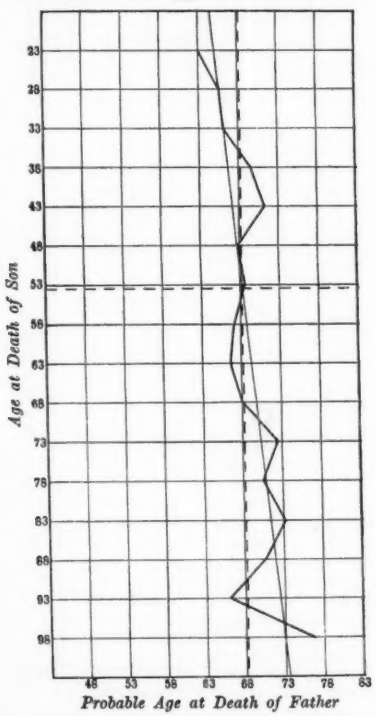
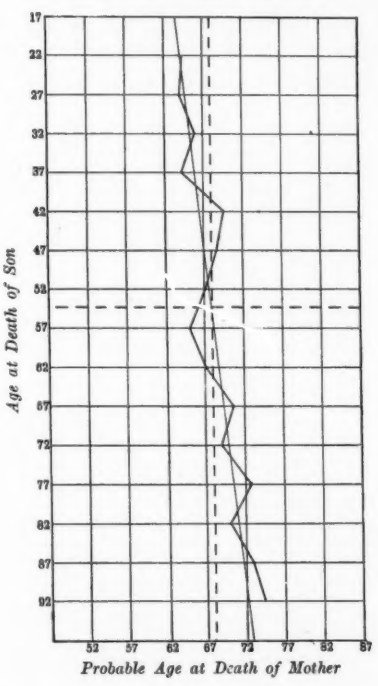


FIG. 4.

Regression Line: Mothers on Sons.



* The reader must disregard the results for ages 93 and 98, for they are based on very few cases.

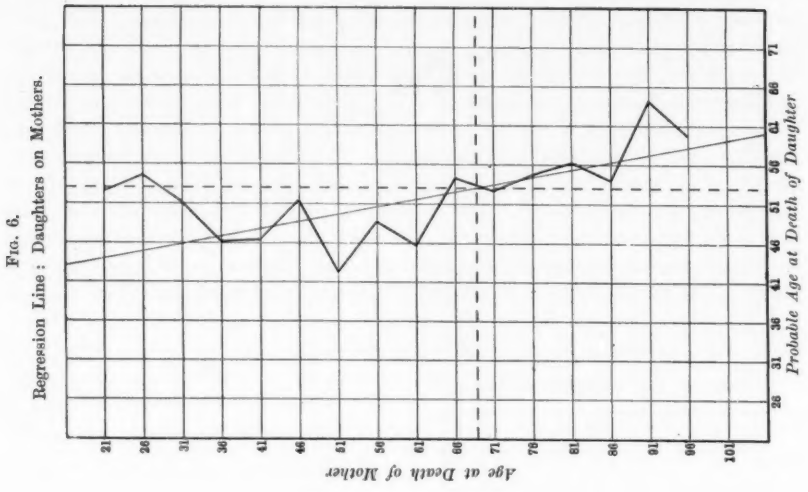


FIG. 6.
Regression Line: Daughters on Mothers.

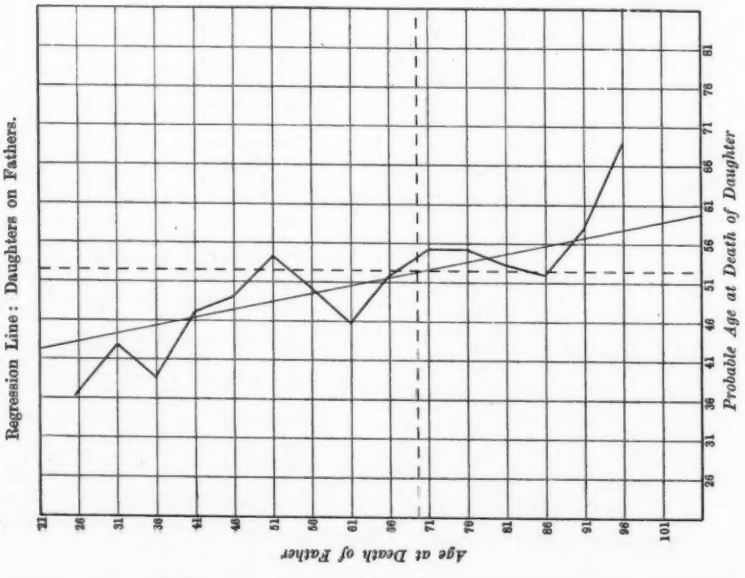


FIG. 5.
Regression Line: Daughters on Fathers.

FIG. 8.
Regression Line: Sisters on Sisters.

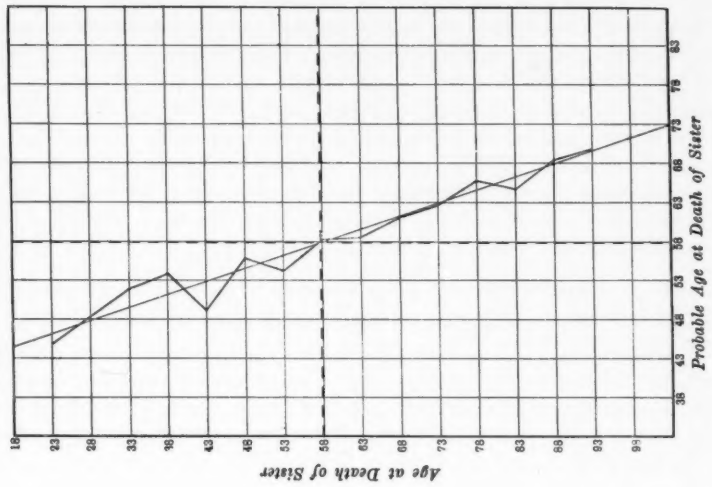
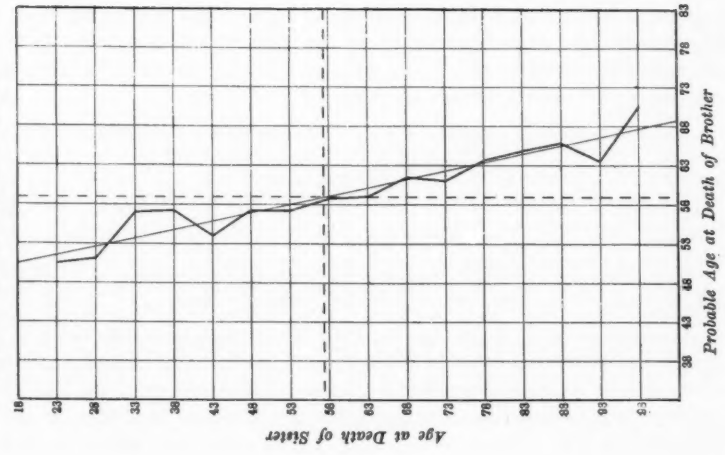


FIG. 7.
Regression Line: Brothers on Sisters.



this statement, however, must be confined to the parental relationship, for even in deaths in middle life we find a fairly continuous and unbroken slope for the regression line in the case of brothers or sisters: see for example Figs. 7 and 8. The problem is, of course, immensely complex and probably rendered especially so by a mixture of different classes of causes, i.e. inheritance of tendencies to develop one or more mortal diseases at definite ages in life, and inheritance of general physical weakness or physical robustness tending merely to shorten or lengthen life as a whole, and largely influenced by environment as far as the definite age at death is concerned. All we can do, in default of special statistics stating the cause of death for each individual, is to examine in broad outline the general influence on the duration of life of an individual of the ages at death of his or her nearest relatives.

(7) To illustrate the use of equations like (1) to (52) the following problems may be considered.

Illustration (i).—*A*'s age is 44, his father and mother are alive at the ages of 79 and 74. His paternal grandfather died aged 69 and his paternal grandmother at 82; his maternal grandfather and grandmother lived to be 81 and 59 respectively. What are the expectations of life of his parents, and what is his own expectation of life based on theirs?

Now this is by no means so straightforward a problem as it might appear at first sight, and there is more than one way of looking at it which will give a fairly reasonable solution. We cannot apply equations (3) and (6) straight off, because the M and W there are a man and woman of the general population, but our man and woman belong to a select class, namely those who live to have adult sons. The average ages of such according to our Table A are: for father 68·370 years and for mother 67·947 years. Hence the probable age at death of *A*'s father and mother would be given by the equations:

$$\begin{aligned} A\text{'s father's age} &= 68{\cdot}370 + {\cdot}1802\,(69 - 68{\cdot}370) + {\cdot}1486\,(82 - 67{\cdot}947) \\ &= 70{\cdot}571 \text{ years.} \\ A\text{'s mother's age} &= 67{\cdot}947 + {\cdot}2057\,(81 - 69{\cdot}547) + {\cdot}1914\,(59 - 68{\cdot}702) \\ &= 68{\cdot}446 \text{ years.} \end{aligned}$$

Thus while *A*'s father gains about 2·2 years by his ancestry, *A*'s mother gains only ·5 years because her mother died comparatively early.

Again, we must not in calculating Σ for the selected class take the σ of sons in general, but rather the value 14·6974 years of fathers of adult sons, and similarly we must take 16·9033 years for the σ from which we calculate the Σ of *A*'s mother's class. This leads us to the values 14·4339 and 16·5677. Or, *A*'s father belongs in an array of men, whose mean age at death is 70·571 years, and whose standard deviation is 14·4339 years. Similarly, *A*'s mother belongs to an array of women whose mean age at death is 68·446 years, and whose standard deviation is 16·5677.

Now A's father has already lived $79 - 70{\cdot}571$ years $= 8{\cdot}429$ years beyond the mean age, and A's mother $74 - 68{\cdot}446$ years $= 5{\cdot}554$ years beyond the mean age. We require their expectations of life. In default of better hypotheses we take the distribution of the array to be represented by the normal curve of errors. Then if X_1 be A's father's further expectation of life, and $x_1 = 8{\cdot}429$ years, $\sigma_1 = 14{\cdot}4339$ years,

$$8{\cdot}429 + X_1 = \frac{\frac{1}{\sqrt{2\pi}}\int_{x_1}^{\infty} x e^{-x^2/(2\sigma_1^2)}\,dx}{\frac{1}{\sqrt{2\pi}}\int_{x_1}^{\infty} e^{-x^2/(2\sigma_1^2)}\,dx},$$

$$= \sigma \times \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x_1'^2} \Big/ \frac{1}{\sqrt{2\pi}}\int_{x_1'}^{\infty} e^{-\frac{1}{2}x_1'^2}\,dx'.$$

These values are to be found from the usual tables, and lead to

$$X_1 = 17{\cdot}364 - 8{\cdot}429,$$
$$= 8{\cdot}935.$$

Thus A's father will probably live to be 87·935 years. Working out the case of A's mother in the same manner we find for her probable further length of life 11·388 years, or her probable age at death is 85·388 years.

If we turn to Ogle's expectation of life table we find the expectation of life of a man of 79 to be 5·07 years, and of a woman of 74 to be 7·25 years, or they will probably reach ages of 84·07 and 81·25 years respectively. In other words A's parents appear to have 4 years' greater duration of life than is deducible from Ogle's results. How is this to be accounted for? I think in the following way. The mean age of all our series of fathers is 66·334 years, but the mean age of fathers with an adult son is 68·370 years. Hence if we consider that the majority of men who live to be 79 are either fathers or in general quite equal to fathers in physique, it follows that A's father increases his expectation of life by 4 years over Ogle's estimate in two nearly equal parts, 2 years because he has an adult son and 2 years on account of the longevity of his ancestry. Treating the case of the mother in the same way we find the mean age at death of mothers of any kind to be 63·225 years, but the age of mothers with adult sons is 67·947 years; in other words about 4·7 years' difference in the duration of life. As A's mother only gains about ·5 years from ancestry, we might expect a total gain of about 5 years on Ogle's estimate. There is actually a gain of a little over four years*. Thus, allowing for the fact that we are selecting special classes, our results are not widely divergent from what we might expect from Ogle's general mortality table.

* The average age at death of our Friends series is 59·97 for adult males and 58·78 for adult females. Ogle's results give about 59·5 for adult males and 62 for adult females. So that our males are longer and not shorter lived than the females.

Turning now to the case of A himself, we shall use his parents' probable lives, 87·935 and 85·388 years, as if they were their actual ages at death*, but what class are we to suppose him to belong to? Does he belong to the mean population of adult sons? If so his class's mean age is 53·490 years. Actually he is a father, without having lost a child, or without any of his children reaching the age of 21. Hence all we can do is to class him under the general group of fathers with a mean age of 66·334 years. Thus we have

A's probable age at death

$$= 66{\cdot}334 + {\cdot}1802\,(87{\cdot}935 - 68{\cdot}370) + {\cdot}1486\,(85{\cdot}388 - 67{\cdot}947)$$
$$= 72{\cdot}748 \text{ years.}$$

Now the mean S.D. of all fathers = 14·8472, and therefore the S.D. of fathers with selected parents = 14·5810.

Thus A belongs to an array of men who die at the average age of 72·748 years with a standard deviation σ_1 of 14·5810 years.

* This assumption is not strictly legitimate for the reasons given in the next illustration. If the mean age of death of fathers be m_1 and of mothers m_2, and the father die at m_1+x and the mother at m_2+y, the probable age at death of the son would be found from an equation of the form given above in Table B to be: $m_3 = c_1 + c_2 x + c_3 y$. Hence the expectation of life $E_{x,\,y}$ of a man aged $m_3 - a_3$ whose parents die at m_1+x and m_2+y would be

$$E_{x,\,y} = \frac{\Sigma^2}{\sqrt{2\pi}} e^{-\frac{(m_3-a_3)^2}{2\Sigma^2}} \Big/ \frac{1}{\sqrt{2\pi}} \int_{-(m_3-a_3)}^{\infty} e^{-\frac{X^2}{2\Sigma^2}}\, dX.$$

Now if the parents be alive at m_1+a_1 and m_2+a_2 years their chances of dying between m_1+x and m_2+y and $m_1+x+\delta x$, $m_2+y+\delta y$ will be

$$\frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2\sigma_1^2}} dx \Big/ \frac{1}{\sqrt{2\pi}} \int_{a_1}^{\infty} e^{-\frac{x^2}{2\sigma_1^2}} dx \quad \text{and} \quad \frac{1}{\sqrt{2\pi}} e^{-\frac{y^2}{2\sigma_2^2}} dy \Big/ \frac{1}{\sqrt{2\pi}} \int_{a_2}^{\infty} e^{-\frac{y^2}{2\sigma_2^2}} dy$$

respectively.

Multiply the product of these chances by the expectation of life $E_{x,\,y}$; we have on integrating for x and y from a_1 to ∞ and from a_2 to ∞ respectively, the total value E of the man's expectation of life:

$$E = \frac{\dfrac{\Sigma^2}{\sqrt{2\pi}} \dfrac{1}{2\pi} \displaystyle\int_{a_1}^{\infty}\int_{a_2}^{\infty} e^{-\frac{(m_3-a_3)^2}{2\Sigma^2}} e^{-\frac{x^2}{2\sigma_1^2}} e^{-\frac{y^2}{2\sigma_2^2}}\, dx\, dy}{\dfrac{1}{\sqrt{2\pi}} \displaystyle\int_{-(m_3-a_3)}^{\infty} e^{-\frac{X^2}{2\Sigma^2}} dX} \Big/ \frac{1}{\sqrt{2\pi}} \int_{a_1}^{\infty} e^{-\frac{x^2}{2\sigma_1^2}} dx \times \frac{1}{\sqrt{2\pi}} \int_{a_2}^{\infty} e^{-\frac{y^2}{2\sigma_1^2}} dy.$$

Here m_3 is the function of x and y given above, Σ is the standard deviation of sons of selected parents, σ_1 and σ_2 the standard deviations of fathers and mothers of adult sons. Without using troublesome quadrature formulæ I do not at present see how to evaluate the integral in the numerator. It certainly deserves investigation.

Since A is 44 years of age, he is $x_1 = 28{\cdot}748$ years before the probable age at death of his class. His expectation of life is accordingly $28{\cdot}748 + X_1$ where

$$X_1 = \frac{\frac{1}{\sqrt{2\pi}}\int_{-x_1}^{\infty} xe^{-x^2/(2\sigma^2)}\,dx}{\frac{1}{\sqrt{2\pi}}\int_{-x_1}^{\infty} e^{-x^2/(2\sigma^2)}\,dx},$$

$$= \frac{\frac{1}{\sqrt{2\pi}}\int_{+x_1}^{\infty} xe^{-x^2/(2\sigma^2)}\,dx}{1 - \frac{1}{\sqrt{2\pi}}\int_{+x_1}^{\infty} e^{-x^2/(2\sigma^2)}\,dx}.$$

Calculating this from the tables in the usual way we find:

$$X_1 = {\cdot}854 \text{ years.}$$

Thus while A gains $6{\cdot}414$ years from his ancestry he only gains ${\cdot}854$ years from the fact that he has already survived 44 years. His probable duration of life is $73{\cdot}60$ years. Ogle's table would give a man of 44, $22{\cdot}7$ years' expectation of life; from our data A has $29{\cdot}6$ years. The difference of 7 years is substantially due to A's good ancestry.

Illustration (ii).—A man of 50 and his wife of 44 years of age have had two children, one of whom, a daughter, died at 8, and the other, a son, at 12. The man wishes to provide an annuity for his wife, if she survives him. What is her expectation of life as a widow? By Equation (15) of Table B the man belongs to a group of men who die at the mean age of $65{\cdot}717$ years, with a standard deviation of $15{\cdot}2830$. By Equation (25) the woman belongs to a group of women who die at the mean age of $61{\cdot}004$ years, with a standard deviation of $17{\cdot}9290$. An erroneous solution of this problem might be obtained in the following manner: the man will most probably live $15{\cdot}717$ years, if he died at the most probable age of death; his widow if she lived to her most probable age of death would be $59{\cdot}717$ years old when he died and have $1{\cdot}287$ years still to live. This is very far indeed from her expectation of widowhood. Such a solution fails because the probable ages of death of certain arrays of men and women do not determine the expectations of life of men and women who have already lived to certain definite ages. Working out as in Illustration (i) the expectations of life of the man and woman, we find them to be $19{\cdot}953$ and $22{\cdot}510$ years respectively*. Would it be correct to say then that the expectation of widowhood is $2{\cdot}557$ years? This again is incorrect; the years of widowhood are in each case correlated with the age at which the husband dies, and we cannot find the mean value of these years from the mean ages of death of husband and wife.

The problem is considerably more complex and we must proceed as follows: Let x_1 be the number of years before or after $65{\cdot}717$ years at which the husband

* Ogle's life table gives $18{\cdot}93$ and $24{\cdot}72$ respectively. Our man and woman, however, do not represent the general population. They have been father and mother and have lost two children.

dies, then the wife, if alive, will be $x_2 = x_1 - 1{\cdot}287$ years from 61·004 years, the probable age at death of the array of such women. Hence the chance that the woman survives her husband if he dies at x_1:

$$= \frac{\frac{1}{\sqrt{2\pi}} \int_{x_2}^{\infty} e^{-x^2/(2\sigma_2^2)}\, dx}{\frac{1}{\sqrt{2\pi}} \int_{b}^{\infty} e^{-x^2/(2\sigma_2^2)}\, dx},$$

where $b = -17{\cdot}004$ and $\sigma_2 = 17{\cdot}9290$.

If she does survive him, her expectation of life

$$= \frac{\frac{1}{\sqrt{2\pi}} \int_{x_2}^{\infty} x e^{-x^2/(2\sigma_2^2)}\, dx}{\frac{1}{\sqrt{2\pi}} \int_{x_2}^{\infty} e^{-x^2/(2\sigma_2^2)}\, dx}.$$

Hence her expectation of widowhood, if the man dies at x_1 expressed in years, is the product of these expressions,

$$= \frac{\frac{1}{\sqrt{2\pi}} \sigma_2^2\, e^{-x_2^2/(2\sigma_2^2)}}{\frac{1}{\sqrt{2\pi}} \int_{b}^{\infty} e^{-x^2/(2\sigma_2^2)}\, dx}.$$

Now the man's chance of dying between x_1 and $x_1 + dx_1$

$$= \frac{\frac{1}{\sqrt{2\pi}} e^{-x_1^2/(2\sigma_1^2)}\, dx_1}{\frac{1}{\sqrt{2\pi}} \int_{a}^{\infty} e^{-x^2/(2\sigma_1^2)}\, dx},$$

where $a = -15{\cdot}717$ and $\sigma_1 = 15{\cdot}2830$.

We now multiply the woman's expectation of widowhood if the man's death takes place at a particular age by the man's chance of dying at that age and integrate for all possible ages of the man at death. We thus find if E_w be the expectation of widowhood of the wife:

$$E_w = \frac{\frac{1}{\sqrt{2\pi}} \int_{a}^{\infty} e^{-x_1^2/(2\sigma_1^2)} \frac{1}{\sqrt{2\pi}} \sigma_2^2\, e^{-x_2^2/(2\sigma_2^2)}\, dx_1}{\frac{1}{\sqrt{2\pi}} \int_{a}^{\infty} e^{-x^2/(2\sigma_1^2)}\, dx \times \frac{1}{\sqrt{2\pi}} \int_{b}^{\infty} e^{-x^2/(2\sigma_2^2)}\, dx}.$$

Here $x_2 = x_1 - h$, where $h = 1{\cdot}287$ years.

Now let us write the subject of integration in the numerator in the form

$$e^{-\frac{1}{2}\frac{1}{\sigma_0^2}(x_1 - c)^2 - \frac{1}{2}f^2}$$

or

$$\frac{1}{\sigma_0^2}(x_1 - c)^2 + f^2 = \frac{x_1^2}{\sigma_1^2} + \frac{x_1^2}{\sigma_2^2} - \frac{2hx_1}{\sigma_2^2} + \frac{h^2}{\sigma_2^2}.$$

We find

$$\sigma_0 = \frac{\sigma_1\sigma_2}{\sqrt{\sigma_1^2+\sigma_2^2}}, \quad c = \frac{\sigma_1^2}{\sigma_1^2+\sigma_2^2}h, \quad f = \frac{h}{\sqrt{\sigma_1^2+\sigma_2^2}}.$$

Thus if $X = x_1 - c$ we have

$$E_w = \frac{\sigma_2 \frac{\sigma_2}{\sqrt{2\pi}} e^{-\frac{1}{2}\frac{h^2}{\sigma_1^2+\sigma_2^2}} \times \frac{1}{\sqrt{2\pi}} \int_{a-c}^{\infty} e^{-\frac{1}{2}\frac{X^2}{\sigma_0^2}} dX}{\frac{1}{\sqrt{2\pi}} \int_a^{\infty} e^{-x^2/(2\sigma_1^2)} dx \times \frac{1}{\sqrt{2\pi}} \int_b^{\infty} e^{-x^2/(2\sigma_2^2)} dx}.$$

Put

$$a' = \frac{a}{\sigma_1}, \quad b' = \frac{b}{\sigma_2}, \quad c' = \frac{a-c}{\sigma_0},$$

and we have

$$E_w = \frac{\sigma_0 \frac{\sigma_2}{\sigma_1} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}\frac{h^2}{\sigma_1^2+\sigma_2^2}} \times \frac{1}{\sqrt{2\pi}} \int_{c'}^{\infty} e^{-\frac{1}{2}X'^2} dX'}{\frac{1}{\sqrt{2\pi}} \int_{a'}^{\infty} e^{-\frac{1}{2}x'^2} dx' \times \frac{1}{\sqrt{2\pi}} \int_{b'}^{\infty} e^{-\frac{1}{2}x'^2} dx'},$$

where

$$h/\sqrt{\sigma_1^2+\sigma_2^2} = \cdot 054{,}629, \qquad c' = -1\cdot 397{,}888,$$
$$a' = -1\cdot 028{,}396, \qquad b' = -\ \cdot 948{,}408.$$

All parts of the above expression can now be found from tables of the ordinates and areas of the normal curve of errors. We find

$$E_w = 13\cdot 6442 \frac{\cdot 398{,}302 \times \cdot 918{,}924}{\cdot 848{,}113 \times \cdot 828{,}538}$$
$$= 7\cdot 107 \text{ years.}$$

Thus the wife has an expectation of a little more than seven years of widowhood, i.e. this is the mean period of widowhood of wives of her class. This is nearly six times the period found for the difference between the probable ages at death of husband and wife and three times the difference between their expectations of life. The whole problem of the expectation of widowhood seems deserving of treatment on general lines.

(8) We may now return to the bearing of our results on the problem of evolution. A very considerable series of investigations on a variety of organs and characters in man have given quite definite results as to both parental and fraternal inheritance. Whether we take mental or physical characters, the parental correlation lies between ·3 and ·5, some of our best results tending nearer to the latter limit; for fraternal correlation the limits appear to be closer, from ·4 to ·5. We may take as typical numbers ·4 for parents and ·45 for brothers. Now it is at once obvious that for duration of life treated as an inheritable character we have got results which fall far below these values. The direct explanation of this lies in the existence of a non-selective death-rate. Hence

the reduction in the correlation of heredity as given in this case will enable us to find approximately the proportions of the selective and non-selective death-rates. As we have already pointed out, the case of brethren is better suited than the case of parent and offspring to test the proportions of the selective and non-selective death-rates.

Now if we take parents and adult children, the average correlation from Table A for all combinations of sexes is ·1365. For adult brethren, including brothers and sisters, it is ·2831. If pN be the number of deaths in N cases which are non-selective, i.e. do not depend in any way on the constitution of the individual, then $(1-p)$ will represent the chance of any individual dying under the selective death-rate, and $(1-p)^2$ the chance that both the members of a pair so die. Hence the ratio of the selective death-rate cases to the whole number of such cases is $(1-p)^2N$ to N, or if r be the observed correlation, and r_0 the correlation to be expected from the laws of inheritance, we should expect $r/r_0 = (1-p)^2$*. We have therefore:

$$\cdot1365 = (1-p)^2\, r_0,$$
$$\cdot2831 = (1-p)^2\, r_1,$$

where r_0 and r_1 are the parental and fraternal correlations. The following table is based upon these equations; we have given a range of values to r_0 and r_1 covering the actually observed numbers for other characters.

TABLE C.

Selective Death-Rate in Man deduced from Ages at Death of Relatives.

Parental Correlation			Fraternal Correlation		
Value	Selective Death-Rate	Non-Selective Death-Rate	Value	Selective Death-Rate	Non-Selective Death-Rate
·3	67·5 p.c.	32·5 p.c.	·4	84·1 p.c.	15·9 p.c.
·4	58·4 p.c.	41·6 p.c.	·45	79·3 p.c.	20·7 p.c.
·45	55·1 p.c.	44·9 p.c.	·5	75·2 p.c.	24·8 p.c.

This table is in good agreement with the general results reached in the earlier paper, namely, that judging from parental inheritance the selective death-rate is 63 to 70 per cent., and from fraternal inheritance about 80 per cent. of the total death-rate †. The drop between the results derived from fraternal and parental correlation is due, as we have before remarked, to the environment changing much more from parent to offspring than it does from brother to brother. Even the environment of two sisters is usually much more alike than that of two

* *Phil. Trans.* A, Vol. 192, p. 277.

† *R. S. Proc.*, Vol. 65, p. 293.

brothers who frequently leave earlier the parental roof, often go much further and are more liable to accident, i.e. to non-selective death. Compare the correlation ·3322 of two sisters with the ·2853 of two brothers.

Look at the matter as we will it is impossible to place the selective death-rate at lower than 50 per cent., and in all probability with the same environment it is over 80 per cent.

(9) The following is, we believe, a fairly accurate statement of the general principle of evolution by natural selection combined with heredity:

(*a*) With a given environment individuals having certain constitutions are fitter to survive than others.

(*b*) They thus have a greater opportunity of reproducing themselves and rearing their offspring.

(*c*) Since the parents' characters are transmitted to their offspring the prevailing characters of the general population will in every case either be continuously modified or are only maintained stable owing to natural selection.

If (*a*) were not true the death-rate would be non-selective. We have shown in this paper that at least 50 and probably 80 per cent. of the death-rate in the case of man is certainly selective. Relatives—i.e. men with similar constitutions—have durations of life substantially correlated. In the case of civilised man the selection is largely due to the struggle with the physical environment, and not to the struggle of individual with individual. We should expect therefore the intensity of the selective death-rate in his case to be smaller than it is for many types of wild life.

In a paper published in the Royal Society's *Proceedings** we have shown that the number of offspring is directly related to the duration of life. We have thus quantitatively demonstrated the truth of (*b*).

Finally, in a fairly long series of papers only in part as yet published it has been proved that physical and intellectual characters are inherited from the parent; (*c*) is incontestably true.

I think, therefore, that we can no longer talk of natural selection as an hypothesis. It is in the case of man demonstrably at work either changing in a quantitatively definite manner his constitution as a whole or else necessary to keep that constitution stable. It is not now correct to say as Lord Salisbury said in 1894 of natural selection: "No man, so far as we know, has ever seen it at work." It is sensibly and visibly at work; a factor in 50 to 80 per cent. of the deaths in the case of man is not a slight perturbation, which we must seek with very refined analysis as the astronomer might seek for a small

* *Data for the Problem of Evolution in Man*, V. On the correlation between Duration of Life and Number of Offspring, Vol. 67, p. 159.

secular acceleration. It is something we run up against at once, almost as soon as we examine a mortality table. But if natural selection be at work and can be seen at work we are still far from the end of our researches. We have got beyond the stage of those who still persist in speaking of natural selection as a hypothetical source of change, but admitting once for all that it is continuously modifying races, we have still to investigate whether it can effectively differentiate races, i.e. be the source of the origin of species. This, whether we attack it from the standpoint of wild life or of laboratory experiment, is again a problem in statistics. Its solution depends on the relative fertility of different sections of a race with each other and with themselves. It cannot possibly be answered by observation of isolated cases, or by general reasoning. As we have said elsewhere: *sine numero nihil demonstrandum est.*

(10) We may note lastly one or two general points bearing on the laws of inheritance which arise from a consideration of Table A.

In the first place inheritance is stronger in members of the same sex, e.g. the correlation in the duration of life is greater between two brothers or two sisters, than between brother and sister; between father and son and mother and daughter, than between father and daughter or mother and son. The daughter seems to be more closely related in duration of life to her parents than the son. This is somewhat opposed to the result obtained for eye-colour*; but is I think quite explicable if we note how much more liable the male is to accidental death; thus the non-selective death-rate is intensified in his case. It is remarkable how much influence the death of a minor daughter has on the expectation of life of the mother as compared with that of the death of a minor son. One would have suspected that the higher mortality in childbed in the latter case would have made the reverse true.

Finally, I may draw attention to the fact that the women are sensibly more variable in their age at death than the men. This confirms the general conclusion reached elsewhere† as to the greater variability of women.

(11) *Summary of Results.*

(*a*) Material for the inheritance of duration of life ought to be collected on a much larger scale, e.g. by assurance offices, than is possible for two isolated workers. We only publish our data as the best available at present.

(*b*) Such material is of great interest, not only for actuarial but for biological problems.

(*c*) Of actuarial interest we may especially note the results:

(i) That the elder members of a family live sensibly longer than the younger.

* *Phil. Trans.*, Vol. 195, p. 117.

† Pearson, *Chances of Death*, Vol. I. p. 373.

(ii) That the expectation of life is seriously modified by either the ages of death of relatives or their present ages*, and can be quantitatively determined so soon as the latter are known.

(iii) The expectation of widowhood must be based not only on the present age of the husband, or on the age of relatives, but on the age at death of children.

(*d*) Of biological interest are:

(i) The possible correlation of a character or organ in a member of a family with his or her birth order.

(ii) The estimate of the intensity of the selective death-rate in man.

(*e*) Natural selection is manifestly at work in man, and is a factor in 50 to 80 per cent. of the deaths which occur.

We must place here a record of our great appreciation of the generous help we have received from Dr Alice Lee in the verification of the numerical work on the tables, and from Mr Karl Tressler in the preparation of the diagrams and the recalculation of the averages on which they are based.

* Thus that a man's father died at 80 is significant for his expectation of life, but it is also significant, if his father is alive at 75.

TABLE I. BROTHERS.

Difference in Duration of Lives in Years. Elder Brother—Younger Brother.

Years' interval between births.	Negative																		Positive															No. of Cases		
	88	*83*	*78*	*73*	*68*	*63*	*58*	*53*	*48*	*43*	*38*	*33*	*28*	*23*	*18*	*13*	*8*	*3*	*3*	*8*	*13*	*18*	*23*	*28*	*33*	*38*	*43*	*48*	*53*	*58*	*63*	*68*	*73*	*78*	*83*	*88*	Totals	Younger lives longer	Elder lives longer
1					3	5	3	2	3	2	2	3	1	5	5	4	8	20	17	5	6	2	6	3	2	2	2	2	2	1	1	1					117	66	51
2			2	2	3	2	3	7	1	8	8	15	12	11	8	17	23	46·5	57·5	23	13	17	16	10	7	10	3	12	8	7	2	2	2		1	1	360	168·5	191·5
3		2		2	2	2	1	1	3	1	5	5	2	1	6	6	8	31	42	13	11	12	2	7	6	4	3	5	2	3	4	3		2			197	78	119
4		1			1	1	2	2	3	3	6	10	6	3	17	6	14	28·5	29·5	12	6	7	13	8	17	5	3	2	2	1	5	2	1		1		218	103·5	114·5
5				3	3	2	3	1	2	1	7	7	1	3	7	7	7	22·5	36·5	15	12	7	8	3	6	4	2	7	1	1	2	5	1	1			188	76·5	111·5
6				1		3	1	2	4	3	2	4	3	3	6	4	12	19·5	36·5	13	8	10	8	7	2	4	4	2	2	3	1	3	1	1			173	67·5	105·5
7				2	1		1	2	3	5	1	5	6	3	7	4	7	20·5	21·5	9	7	8	5	3	6	6	1	3	5	6	4	1	2	3			158	67·5	90·5
8		1		2	3	2		1	4	5		6	9	1	8	4	2	9	17·5	18·5	11	6	9	7	6	3	2	4	4	2	5	1	2	1	1	1	158	74·5	83·5
9	2			1			3	2	1	4	3	1	1	4	5	4	10	14	19	7	6	10	6	4	7	1	4	6	3		1	2		1			132	55	77
10			1		1	1		1		2		1	1	3	6	4	5	13·5	12·5	12	3	8	8	3	3	1	1	5	6	2	3	2			1		110	39·5	70·5
11			1		1	2		1	2		1		1	3	1	4	3	6·5	9·5	7	5	8	2	6	3	4	3	2	1	2	1	2	1			83	26·5	56·5
12				1		1		1	2		1		3	3	2	4	3	9·5	12·5	6	5	5	3	4	4	1	1	2	1	3	1						79	30·5	48·5
13				2	1		1		1	1	1	1	1	2	2	3	3	1·5	4·5	8	5	5	2	2		1	1	2	2	3	1	1	1		1		60	20·5	39·5
14										1	1		2		1	1	3	3	9	4	6	3	4		5	2		2	1	1	3		1	1			54	12	42
15					1								2		2	2	1	4·5	7·5	4	1	1	3	2	3	1	1	1	2	2	2	1					44	12·5	31·5
16								1				1	2	1	1		1	6	5	2	1	2	1			2	3	2		2			1				34	13	21
17		1												1	1	2		1·5	2·5	1	1	1		2	2	2	1		2								21	6·5	14·5
18									1		1	1		1		2	2	1	2	1				1	1			1		1			1				17	9	8
19				1					1	1			2			1	2	1	1	2	1	4	1			1					2		1				22	9	13
20				1	1												1	2					1	1							1	1					9	5	4
21																										1											1		1
22																		·5	·5		1			1	1	1			1								7	·5	6·5
23																									1												1		1
24														1							1											1						3	1	2
25																																					0		
26																		1																			1	1	
Totals	2	5	4	18	21	21	19	27	32	32	45	63	47	56	81	77	122	271·5	344·5	155	105	123	96	73	79	54	39	60	40	45	35	28	13	12	6	1	2247	943·5	1303·5

TABLE II. SISTERS.

Difference in Duration of Lives in Years. Elder Sister—Younger Sister.

Years' interval between births.	Negative																			Positive																				No. of Cases		
	93	*93*	*88*	*83*	*78*	*73*	*68*	*63*	*58*	*53*	*48*	*43*	*38*	*33*	*28*	*23*	*18*	*13*	*8*	*3*	*3*	*8*	*13*	*18*	*23*	*28*	*33*	*38*	*43*	*48*	*53*	*58*	*63*	*68*	*73*	*78*	*83*	*88*	*93*	Totals	Younger lives longer	Elder lives longer
1			2	1	1	2	2	1	2		1	2		2	3	1	3	2	7	9·5	11·5	3	6	5	2	3	3		1	2	1	1	1	1		1					82	41·5	40·5
2		1		1	1	5	2	2	2	5	2	8	2	4	3	8	11	10	21	30·5	33·5	15	12	6	10	13	2	2	6	4	2	6	1	1	2		1			235	118·5	116·5
3			1	1					3	1	2	3	2	1	2	5	4	7	9	27·5	27·5	15	12	7	8	6	4	4	2	5	1	3								163	68·5	94·5
4				1	1		1	3	2	4	1	1	5	3	3	5	7	6	9	20	22	9	9	4	3	4	3	3	3	1	2	1		2	1	2	2		1	142	70	72
5			3			1		1	3	4	2	1		7	4	2	4	5	9	21	25	14	5	12	6	4	2	3	3		3		1		1					146	67	79
6			1		1	1	2	1	2	4	3	1	1	2		5	5	6	11	18·5	23·5	10	10	8	2	5	3	3	6	2	3	2	2		1		1			146	64·5	81·5
7						1	1				2	4	2	1	5	4	9	6	5	8·5	5·5	6	11	10	3	7	4	4	1	2		3	1	2						108	48·5	59·5
8				1			1	1		1	2	2		1	3	2	1	3	4	11	12	10	8	5	10	2		3	2	1	3			3		1				93	33	60
9		1				1	3		2	1	2	1		3	1	2	5	6	4	8	12	12	2	3		4	3	1	1	2	1	1	1	1						84	40	44
10									1	1	1	1	1	1		2		3	3	2·5	10·5	8	7	4	3	4	1	3	1	2	1	1	4	1		1				68	16·5	51·5
11						1	1			1		2		1	4	2	2	2	3	10·5	10·5	1	7	7	1	5		1	1	1	2	2			1	2				72	29·5	42·5
12	1					1		1	1	1					3		2		3	6	6	2	6	3	1	3	2	4	3	4	1								54	19	35
13								1		1	1				3	2		1	6	6	5		5	6	4	2	2	1	1	1	1		1			1			51	21	30
14										1	3					1	1	5	2	5	5	3	2	2	2	1	2		2		2			1					40	18	22
15									1							1				1	3	3	2	2	3	1	2		1			1	2							23	3	20
16									1				1				1	1	2	2·5	4·5		2	2	2	2	1	2							1					25	8·5	16·5
17						1								1	1	2	1			·5	·5		2	2	1		2	1	2			1			1					19	6·5	12·5
18										1							1			2·5	3·5	1	1	2	1			1				2								16	4·5	11·5
19											1									2·5	1·5		1	1	1	1	1	2	1				1	1						15	3·5	11·5
20																					2	3		1				1			1									8		8
21																				1		1	2																	4	1	3
22																					1	1	1			1														4		4
23																				1	1											1								3	1	2
24																								1																1		1
25																					1	1																		2		2
26																											1													1		1
27																																									
28																																									
29																										1														1		1
Totals	1	2	7	5	4	14	13	11	20	24	23	26	14	27	35	44	57	63	98	195·5	227·5	118	113	93	64	67	38	40	36	26	24	24	18	12	9	6	6		1	1606	683·5	922·5

TABLE III.

Father and Adult Son.

Duration of Life of Father.

Duration of Life of Adult Son.	*23*	*28*	*33*	*38*	*43*	*48*	*53*	*58*	*63*	*68*	*73*	*78*	*83*	*88*	*93*	*98*	*103*	Totals
23	1	1	2	5	3	11	6	7	11	9	6	12	8	2		2		86
28				1	6	4	5	12	15	10	13	10	7		1	1		85
33			1	2	2	5	7	8	7	10	7	8	8	4	1			70
38		1	1	2		2	8	5	3	9	11	11	9	5	2	1		70
43		1			1	5	1	5	6	11	10	10	17	5				72
48			1	1	2	5	5	4	6	9	12	15	5	3				68
53		1		3		5	7	3	2	11	11	14	10	1	1	1		70
58			1	3		4	5	10	8	10	5	8	9	3		2		68
63		2	1	3	5	1	4	8	13	9	11	11	11	5				84
68			1	6	3	6	7	5	5	6	14	16	12	7	2			90
73		1		2	1	6	5	4	7	9	10	14	13	8	8	1	1	90
78			1	1	2	2	4	4	4	10	5	8	9	4		3		57
83				1	1	5	3	1	2	3	7	10	13	3	2	2		53
88		1				2	3		1	4	7	5	1	2		2		28
93						1				2	2							5
98						1						1		1	1			4
Totals	1	8	9	30	26	65	70	76	90	122	131	153	132	53	18	15	1	1000

TABLE IV.

Father and Minor Son.

Duration of Life of Father.

Duration of Life of Minor Son.	*26*	*31*	*36*	*41*	*46*	*51*	*56*	*61*	*66*	*71*	*76*	*81*	*86*	*91*	*96*	Totals
**0—1*	4	12	17	20	24	32	32	48	56	42	43	39	16	6	3	394
2—3	1		9	10	10	11	10	21	17	21	17	14	10	2	1	154
4—5	1	2	2	4	2	5	4	12	9	11	11	5	3	2		73
6—7	1	1	1	2	2	2	7	5	8	6	7	4	4			50
8—9	2	1	1	1	1	1	3	4	3	5	3	7	4			36
10—11	1	1		3	3	1	5	2	6	3	7	1	1			34
12—13			3	2	1	2	4	3	4	5	8	2	5			39
14—15	1		1	4	3	1	4	1	3	6	9	6	2	1		42
16—17		1				3	2	9	3	1	7	4	3			33
18—19		2	2	1	2	6	4	8	9	7	9	3	3	2	1	59
20—21				2	2	2	2	3	2	3	1	7	4		1	29
Totals	11	20	36	49	50	66	77	116	120	110	122	92	55	13	6	943

* Centred at ·5 years.

TABLE V.

Father and Adult Daughter.

Duration of Life of Father.

Duration of Life of Adult Daughter.	*26*	*31*	*36*	*41*	*46*	*51*	*56*	*61*	*66*	*71*	*76*	*81*	*86*	*91*	*96*	*101*	Totals
21	1	1	4	4	4	6	6	12	17	16	18	11	14	2			116
26	1		3	2	11	8	10	11	15	13	11	9	9	3			106
31	1	1	1	6	1	5	7	6	18	11	13	14	6	2			92
36	1		1	2	3	2	5	7	9	9	13	11	11	2	1	1	78
41		2	2	2	2	2	9	9	7	9	11	8	7	1	1		72
46		1		1	2		6	7	5	7	11	17	2	2	1		62
51		2	2		4	1	2	2	10	7	11	14	8	1			64
56			2		1	4	5	4	5	8	9	10	4	1			53
61		1		1	4	3	8	6	8	6	20	13	7	2	1		80
66			2		4	2	6	6	10	8	24	11	9	2			84
71	1			2	2	5	8	6	19	11	16	11	10	3	1	1	96
76				3	1	2	3	5	14	19	20	16	10	3			96
81					3	7	4	3	10	16	11	9	10	1	6		80
86				1	4	4	3		4	9	10	5	3	4	1		48
91				1		2	1		2	1	5	2	1	1		1	17
96				1		1		1	2	4		1		1		1	12
Totals	5	8	17	26	46	54	83	85	155	154	203	162	111	31	12	4	1156

TABLE VI.

Father and Minor Daughter.

Duration of Life of Father.

Duration of Life of Minor Daughter.	*26*	*31*	*36*	*41*	*46*	*51*	*56*	*61*	*66*	*71*	*76*	*81*	*86*	*91*	*96*	Totals
**0—1*	7	10	16	24	12	22	18	38	38	40	45	39	19	4	2	334
2—3	1	4	6	7	11	9	2	12	20	10	16	7	11	1	1	118
4—5	1	2	4		7	6	7	11	10	4	6	10	1		1	70
6—7	1	3	2		2	6	4	3	11	3	4	5	4		1	49
8—9		1	3			6	5	6	6	1	6	3	2		1	40
10—11					3	3	1	3	5	3	8	2				28
12—13		1		1	3	2		2	4	4	4		3			24
14—15		1	1	5	3	4	2	5	6	2	6	3	3	2		43
16—17			3	1	3	2	6	4	4	3	8	4	5	1		44
18—19	1		4	2	4	7	2	9	11	6	8	3	4	1	1	63
20—21		1	2	2		3	1	3	3	8	6	1	9	1		40
Totals	11	23	41	42	48	70	48	96	118	84	117	77	61	10	7	853

* Centred at ·5 years.

TABLE VII.

Mother and Adult Son.

Duration of Life of Mother.

Duration of Life of Adult Son.	*22*	*27*	*32*	*37*	*42*	*47*	*52*	*57*	*62*	*67*	*72*	*77*	*82*	*87*	*92*	*97*	Totals
22		1	3	4	7	7	1	10	7	12	5	9	7	7	5	1	86
27	1	1	1	5	6	7	5	6	5	12	8	8	5	8	5		83
32		1	4	3	4	3	5	5	6	7	10	6	14	3	4		78
37		3	4	6	3	1	6	5	4	18	6	10	11	4	4		85
42		1	3	4	2	2	2	4	3	10	10	22	13	2	4	2	84
47		3	2	2	3	3	3	4	5	4	7	16	9	8	4	1	74
52	1		1	5	3	2	4	9	11	10	5	12	10	11	2		86
57			3	4	10	6	6	5	4	11	14	14	13	6	2		98
62		2	3	5	7	6	6	7	8	7	10	10	17	9	11		108
67		2	2	1	9	4	2	3	14	16	9	17	18	22	5	1	125
72	1	1	1	3	5	7	4	4	10	12	7	17	16	13	4	1	106
77	1			3		3	2	3	10	14	9	17	6	16	7	1	92
82		1	4	1	2	5	3	2	4	6	10	10	7	12	6	1	74
87		1				1	1	1	2	5	4	8	2	4	2	1	32
92									1		2	2		1			6
97										1			1				2
102											1						1
Totals	4	17	31	46	61	57	50	68	94	145	117	181	149	126	65	9	1220

TABLE VIII.

Mother and Minor Son.

Duration of Life of Mother.

Duration of Life of Minor Son.	*23*	*28*	*33*	*38*	*43*	*48*	*53*	*58*	*63*	*68*	*73*	*78*	*83*	*88*	*93*	*98*	*103*	Totals
**0—1*	9	13	34	30	20	29	20	27	39	27	34	49	19	16		1	1	368
2—3	4	7	8	23	8	5	12	9	8	21	12	8	7	2	1			135
4—5		2	6	6	7	4	4	7	3	3	4	5	2	2				55
6—7		1	4	2	3	3	4	5	2	2	1	2	4		1	1		35
8—9		1	2	2		2		2	2	2	4	9	2	2	1			31
10—11			2		1	2	1	2		4	4	2	4	1				23
12—13		1	1	2	2	1		2	2		2	2	6					21
14—15	1			1	3	5		2	4	2	3	8		1				30
16—17			1	2	1	1		4	2	2	1		4	1				19
18—19			5	3	5	4	4	1	4	6	5	7	3	2	1			50
20—21		1	2	3	3	3		6	1	1	2	3	1	1		1		28
Totals	14	26	65	74	53	59	45	67	67	70	72	95	52	28	4	3	1	795

* Centred at ·5 years.

TABLE IX.

Mother and Adult Daughter.

Duration of Life of Mother.

Duration of Life of Adult Daughter.	*21*	*26*	*31*	*36*	*41*	*46*	*51*	*56*	*61*	*66*	*71*	*76*	*81*	*86*	*91*	*96*	*101*	Totals
21		1	2	5	7	2	7	5	9	11	9	10	11	8	2		1	90
26		1	3	6	5	6	4	2	10	8	5	13	10	12	2			87
31			3	4	1	4	2	6	3	8	17	12	8	10	3	1		82
36		1	2	5	4	7	4	6	8	4	16	8	13	13				91
41	1	1	2	1	4	2	2	4	7	6	12	12	8	7		1		70
46	1		1		3	1		3	3	6	8	8	11	8	2	1		56
51		1	1	3	2	8	3	2	4	8	6	7	4	4	4		1	58
56			1	2	1	4		2	7	4	6	8	5	4	3		1	48
61		1	1	3	2	4	2	1	4	12	6	12	18	6	4	1		77
66			2	1	2	4	2	5	5	7	14	16	14	7	5			84
71	1		1	2	1	4	3	3	4	13	8	13	14	13	12	3		95
76		3	2	1	3	2	1	3	5	5	12	12	20	13	3			85
81		1	1	3	2	3	1	2	1	11	10	10	14	9	7			75
86			2	2		1		2	1	5	6	6	4	10	3	1		43
91			1		1	2				1	3	4	2	1	2			17
96					1							2	2		1			6
Totals	3	10	25	38	39	54	31	46	71	109	138	153	158	125	53	8	3	1064

TABLE X.

Mother and Minor Daughter.

Duration of Life of Mother.

Duration of Life of Minor Daughter.	*21*	*26*	*31*	*36*	*41*	*46*	*51*	*56*	*61*	*66*	*71*	*76*	*81*	*86*	*91*	*96*	*101*	Totals
**0—1*	7	15	25	35	25	20	21	17	33	15	22	39	19	11	4			308
2—3	1	5	7	14	8	12	11	14	9	14	14	10	12	4	3	2		140
4—5		2	5	6	5	3	9	6	8	2	6	6	3		1		1	63
6—7			3	3	3	4	5	4	2	3	6	4	3	5		1		46
8—9		1	1	1	4	4	2	3	1	1	4	7	2	1				32
10—11		1	1	3	2	3	1	1	1		4		3	2	1			23
12—13				5	2	1	1	3	1		1	2	2	1				19
14—15				2	2	3		2	2	1	5	5	4	4	1			31
16—17		2	1	3	6	2	2	2	1	4	5	4	2	1	4			39
18—19		1	2	3	3	5			5	2	9	4	4	2	2			42
20—21		1		2	5	1	5	2	4	6	3	2	1	5	1		1	39
Totals	8	28	45	77	65	58	57	54	67	48	79	83	55	36	17	3	2	782

* Centred at ·5 years.

TABLE XI.

Adult Brothers (Asymmetrical).

Duration of Life of Elder Brother.

Duration of Life of Younger Brother.	*23*	*28*	*33*	*38*	*43*	*48*	*53*	*58*	*63*	*68*	*73*	*78*	*83*	*88*	*93*	*98*	Totals
23	5	8	7	10	8	5	1	8	4	12	9	2	5	1			85
28	12	9	10	6	6	10	5	2	3	3	6		9	2			83
33	1	5	9	10	6	4	3	7	3	4	2	4	3	1		1	63
38	4		2	6	5	2	3	1	6	7	6		3				45
43	1	3	8	3	4	2	8	7	1	9	6	4	2	2			60
48	3	3	4	9	6	8	3	10	10	9	2	7	5	2			81
53	4	3	5	6	5	3	4	4	4	8	7	4	6	2		1	66
58	6	5	5	5	1	5	6	6	5	13	5	11	8	3	2		86
63		4	8	5	6	7	2	6	8	11	12	7	9	3			88
68	3	2	4	3	3	2	1	8	7	14	16	8	6	6	1		84
73	2		1	9		4	4	3	10	15	20	12	9	2	1	1	93
78	4	1	6	5	4	2	6	8	4	11	4	14	11	5	3	2	90
83	2		4	3	1	2	3	3	1	6	4	6	6	3	1		45
88	1		1	2			1			3	1	7	5	4		1	26
93										3			2				5
Totals	48	43	74	82	55	56	50	73	66	128	100	86	89	36	8	6	1000

TABLE XII.

Adult Brothers (Symmetrical).

Duration of Life of First Brother.

Duration of Life of Second Brother.	*23*	*28*	*33*	*38*	*43*	*48*	*53*	*58*	*63*	*68*	*73*	*78*	*83*	*88*	*93*	*98*	Totals
23	10	20	8	14	9	8	5	14	4	15	11	6	7	2			133
28	20	18	15	6	9	13	8	7	7	5	6	1	9	2			126
33	8	15	18	12	14	8	8	12	11	8	3	10	7	2		1	137
38	14	6	12	12	8	11	9	6	11	10	15	5	6	2			127
43	9	9	14	8	8	8	13	8	7	12	6	8	3	2			115
48	8	13	8	11	8	16	6	15	17	11	6	9	7	2			137
53	5	8	8	9	13	6	8	10	6	9	11	10	9	3		1	116
58	14	7	12	6	8	15	10	12	11	21	8	19	11	3	2		159
63	4	7	11	11	7	17	6	11	16	18	22	11	10	3			154
68	15	5	8	10	12	11	9	21	18	28	31	19	12	9	4		212
73	11	6	3	15	6	6	11	8	22	31	40	16	13	3	1	1	193
78	6	1	10	5	8	9	10	19	11	19	16	28	17	12	3	2	176
83	7	9	7	6	3	7	9	11	10	12	13	17	12	8	3		134
88	2	2	2	2	2	2	3	3	3	9	3	12	8	8		1	62
93								2		4	1	3	3				13
98			1				1				1	2		1			6
Totals	133	126	137	127	115	137	116	159	154	212	193	176	134	62	13	6	2000

TABLE XIII.

Minor Brothers (Symmetrical).

Duration of Life of First Minor Brother.

Duration of Life of Second Minor Brother.	0*	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	Totals
0*	102	53	26	8	13	3	6	5	5	4	8	5	4	3	1	2	1	3	3	8	6	269
1	53	68	29	12	14	4	3	1	4	4	3	1	1	1	1	6	5	3	7	5	3	228
2	26	29	20	7	7	5	1	2	3		3	1	1	2	2	1		3	2	3	3	121
3	8	12	7	2	4	3			1	1	2	3		2	1	2	2		1	3	1	55
4	13	14	7	4	2	1	3	1	2		2	2	1				1		3	2	2	60
5	3	4	5	3	1			1				1						1	1		3	23
6	6	3	1		3				1				1			1		3	1	1		21
7	5	1	2		1	1			1	1	1				1			1		2	1	18
8	5	4	3	1	2		1	1					1					1				19
9	4	4		1				1				1	1	1								13
10	8	3	3	2	2			1						2	1	1				1		24
11	5	1	1	3	2	1				1		2		1					2	2		21
12	4	1	1		1		1		1	1						1		1				12
13	3	1	2	2						1	2	1						1		1		14
14	1	1	2	1				1			1											7
15	2	6	1	2			1				1		1					1		2		17
16	1	5		2	1														1			10
17	3	3	3			1	3	1	1				1	1		1					1	19
18	3	7	2	1	3	1	1					2					1				3	24
19	8	5	3	3	2		1	2			1	2		1		2				2	1	33
20	6	3	3	1	2	3		1										1	3	1	2	26
Totals	269	228	121	55	60	23	21	18	19	13	24	21	12	14	7	17	10	19	24	33	26	1034

* Centred at ·25 years.

TABLE XIV.

Adult Brother and Minor Brother.

Duration of Life of Adult Brother.

Duration of Life of Minor Brother.	23	28	33	38	43	48	53	58	63	68	73	78	83	88	93	98	Totals
0*	21	14	16	17	15	21	17	10	13	16	15	5	6		1		187
1	16	13	19	12	6	9	11	4	13	12	8	4	4	1		1	133
2	6	5	7	4	6	11	5	8	7	6	6	8	5	1		1	86
3		4	5	5	2		4	1		1	2	1	1				26
4	3	4	3	6	2	2	4	2	6	2	1	1	1				37
5	2	2	2	1	1	3	4	1	1	1	1						19
6	5	4	2	2	2	2		2	1	3	1	2		2			28
7		3	1		6			1		3		2		2			18
8	1	5	3	5		1	2	3	1								21
9	4	2	1	1	3		1	1	3	2	2	1					21
10	1	1	2				1	1	2	5	1	2	1				17
11	1		2	3		3	1		2	1							13
12	1		1	1	1	1	1					1		1			8
13	2		1	2	2	1	2		1	1	1						13
14	2	1		1	1	1	1			3				1			11
15	1									3		2					6
16		3		2			2	1		2							10
17	3	1	1	1			2	1	1		3	1	2				16
18	1	5	2	2		2	1		2	1							16
19	7	4	2	4	4	2	1	2	2	3	3	6					40
20	4	5	2	1	5	2	1				2	3	2				27
Totals	81	76	72	70	56	61	61	38	55	65	46	39	22	8	1	2	753

* Centred at ·25 years.

TABLE XV.

Adult Sisters (Asymmetrical).

Duration of Life of Adult Elder Sister.

Duration of Life of Adult Younger Sister.	*23*	*28*	*33*	*38*	*43*	*48*	*53*	*58*	*63*	*68*	*73*	*78*	*83*	*88*	*93*	*98*	Totals
23	20	18	5	10	4	1	6	3	4	10	9	9	6	2			107
28	12	15	6	4	5	9	9	4	8	9	10	6	5	3	2		107
33	4	8	4	3	4	4	1	4	6	4	4	7	2				55
38	6	2	3	6	6	3	1	4	5	4	2	7	7				56
43	3	5	5	3	5	2	3	2	4	3	6	3	1	2			47
48	5	2	6	3	4	3	3	1	7	6	4	4	3	4	1		56
53	3	4	3	2	1		2	3	8	9	5	7	4	1			52
58	2	6	1	2	1	3	6	6	7	7	8	8	5	1	1		64
63	4	4	1	6	2	3	1	7	8	11	8	10	3	5	1		74
68	3	4	8	4	3	4	5	4	14	10	8	22	10	7	3		109
73	2	4	5	2		6	2	3	9	9	15	18	7	4	4		90
78	1	1	4	7	1	3	2	3	4	11	20	24	17	6			104
83	1	4	3	3	2	2		3	4	9	10	12	9	5	1	1	69
88		1		3	1	3			2	6	3	11	5	5	2		42
93						1	1	1	1	1	2	2	3	1	1		14
98				1					1			1		1			4
Totals	66	78	54	59	39	47	42	48	92	109	114	151	87	47	16	1	1050

TABLE XVI.

Adult Sisters (Symmetrical).

Duration of Life of First Adult Sister.

Duration of Life of Second Adult Sister.	*23*	*28*	*33*	*38*	*43*	*48*	*53*	*58*	*63*	*68*	*73*	*78*	*83*	*88*	*93*	*98*	Totals
23	40	30	9	16	7	6	9	5	8	13	11	10	7	2			173
28	30	30	14	6	10	11	13	10	12	13	14	7	9	4	2		185
33	9	14	8	6	9	10	4	5	7		9	11	5				109
38	16	6	6	12	9	6	3	6	11	8	4	14	10	3		1	115
43	7	10	9	9	10	6	4	3	6	6	6	4	3	3			86
48	6	11	10	6	6	6	3	4	10	10	10	7	5	7	2		103
53	9	13	4	3	4	3	4	9	9	14	7	9	4	1	1		94
58	5	10	5	6	3	4	9	12	14	11	11	11	8	1	2		112
63	8	12	7	11	6	10	9	14	16	25	17	14	7	7	2	1	166
68	13	13	12	8	6	10	14	11	25	20	17	33	19	13	4		118
73	11	14	9	4	6	10	7	11	17	17	30	38	17	7	6		204
78	10	7	11	14	4	7	9	11	14	33	38	48	29	17	2	1	255
83	7	9	5	10	3	5	4	8	7	19	17	29	18	10	4	1	156
88	2	4		3	3	7	1	1	7	13	7	17	10	10	3	1	89
93		2				2	1	2	2	4	6	2	4	3	2		30
98				1					1			1	1	1			5
Totals	173	185	109	115	86	103	94	112	166	218	204	255	156	89	30	5	2100

TABLE XVII.

Minor Sisters (Symmetrical).

Duration of Life of First Minor Sister.

Duration of Life of Second Minor Sister.	0*	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	Totals
0*	78	41	22	10	4	14	1	6	5	1		1	1	2	5	3	6	2	2	2	3	209
1	41	34	24	13	13	11	8	8	5	1	2	2	1	3	1	1	5	3	3	4	5	188
2	22	24	14	5	3	4	7	2	4	2	1		1	1	2	1	1	2	1	1	1	99
3	10	13	5	12	2	1		2	2	1	1	1				2	5		1	1	1	60
4	4	13	3	2	2		5		2	1	1				1		1	1	1	1	2	40
5	14	11	4	1				1	1						1				1	2	1	37
6	1	8	7		5			1				1	1					2			1	27
7	6	8	2	2		1	1		2	1	1		1			2		2	1	1	2	33
8	5	5	4	2	2	1		2		1	1				1	2	1	2	1			30
9	1	1	2	1	1			1	1			1							1			10
10		2	1	1	1			1	1			1				1			1	1		11
11	1	2		1			1			1	1								1		1	9
12	1	1	1				1	1								1				1		7
13	2	3	1													1						7
14	5	1	2		1	1			1							1	1	2		2		17
15	3	1	1	2				2	2		1		1	1	1	4		1	3	1		24
16	6	5	1	5	1				1						1		2	1			1	24
17	2	3	2		1		2	2	2						2	1	1			1		19
18	2	3	1	1	1	1		1	1	1	1	1				3			2	1		20
19	2	4	1	1	1	2		1			1		1		2	1		1	1			19
20	3	5	1	1	2	1	1	2				1					1					18
Totals	209	188	99	60	40	37	27	33	30	10	11	9	7	7	17	24	24	19	20	19	18	908

* Centred at ·25 years.

TABLE XVIII.

Adult Sister and Minor Sister.

Duration of Life of Adult Sister.

Duration of Life of Minor Sister.	23	28	33	38	43	48	53	58	63	68	73	78	83	88	93	98	Totals
0*	16	16	7	5	6	7	8	10	14	13	6	13	7	4	1		133
1	16	5	12	8	10	3	6	5	7	7	3	4	5	3			94
2	4	7	9	3	2	3	3	2	1	3	6	8	1	1	1		54
3	2	1			1	3	2	1	3	2		2					17
4	4	2	1	1	3	3		2	1	1		1					19
5	4		2	1	2	1	1			1			1				13
6	9	2	1	1	2			2	3	1	5	2		1			29
7	5		1	1	1		1		2								11
8	1	4	1		1	1		2		3		3	1				17
9		1			1	2		1			1	1					7
10	2	1	1			1	1	1	1	2		1	1	1			13
11	1			1		1			1	1	1	2	1				9
12	2	3		1			1			1							8
13		2		4				1	1	1	2	3					14
14	2		2	1		1	1		1	2		1		2			13
15	1	2				1	1		2	1	2	3		1			14
16	2	2	1	2				1		1	3	1	1				14
17	2	3	2	1	1			1		1							11
18	1		2	2					3				2				10
19	4	2	1	4			2	3		4	1	2	2				25
20	5		2	1					2	2							12
Totals	83	53	45	37	30	27	27	32	42	47	30	47	22	13	2	0	537

* Centred at ·25 years.

TABLE XIX.

Adult Brother and Adult Sister.

Duration of Life of Adult Brother.

Duration of Life of Adult Sister.	*23*	*28*	*33*	*38*	*43*	*48*	*53*	*58*	*63*	*68*	*73*	*78*	*83*	*88*	*93*	*98*	Totals
23	15	12	18	20	6	11	10	5	12	12	10	11	9	2			153
28	14	17	10	19	11	7	13	12	11	10	13	10	7	4			158
33	9	11	9	6	2	6	9	5	8	16	10	17	5	6			119
38	4	7	6	9	9	2	9	8	6	16	5	10	11	1	1		104
43	9	2	5	7	12	7	6	7	8	11	12	2	5	2			95
48	8	5	8	4	6	7	9	13	7	14	18	7	2	5		1	114
53	3	8	6	8	3	5	10	4	8	10	12	12	7				96
58	11	7	3	5	4	7	4	11	9	22	12	15	8	2	1		121
63	11	3	7	7	8	4	6	6	17	21	21	12	4	6			133
68	7	8	11	7	8	13	15	7	22	35	24	27	10	8	1	1	204
73	9	8	6	7	8	7	18	22	23	23	19	18	13	9		3	193
78	12	1	6	13	7	5	16	19	23	27	26	26	27	7	2	2	219
83	4	5	1	2	3	8	11	13	11	15	20	11	19	4	3		130
88		1	2	9	2	1	5	4	9	12	11	6	8	6	3		79
93		3	1		1		1	1	1	2	2	4	2	3			21
98				2						2			2		2		8
Totals	116	98	99	125	90	90	142	137	175	248	215	188	139	65	13	7	1947

TABLE XX.

Minor Brother and Minor Sister.

Duration of Life of Minor Brother.

Duration of Life of Minor Sister.	*0**	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	*19*	*20*	Totals
*0**	50	41	15	8	3	4	6	1		2	6	1		1	1	1		5	2	3	1	151
1	30	41	24	16	5	3	4	2	2	2	4	1	1	5	1		1	2	2	3	4	153
2	15	19	8	4	5	6	3	1	1		2	2	1					2	2	2	2	75
3	7	10	8	2		2	3	2	3	3			2			1	1		1			45
4	7	5	5	1	1	3	2	1	1	1		1				1		2	2		4	37
5	6	9	7	6	1			1	1	2	1								1		1	36
6	3	2	3	4	1	1			1			1							1	2	2	21
7	7	1	1	1		1	3						1		1		1			1		18
8	2	6	3	4	1		1	1	1		1									3		23
9	1	3						1												1	1	7
10	5	5	3		2	1		1												1		18
11	1	2			1		1	1	1													7
12	1	3	2	1											1					1	3	12
13	4	3	1		2	1	2	1		1										2		17
14		1	1									2	1	1					1			7
15	3	1		1		1				1		1										8
16	4	2	2	1		1		2		1		1							1		1	16
17	1	4		1	1				1			1						1		1		11
18	2	1	3					1					1		1				1	1	2	13
19	2	4		1			1	1						1				1				11
20	1			1			1	1												1		5
Totals	152	163	86	52	23	24	27	18	12	13	14	11	7	8	5	3	3	13	14	22	21	691

* Centred at ·25 years.

TABLE XXI.

Adult Brother and Minor Sister.

Duration of Life of Adult Brother.

Duration of Life of Minor Sister.

	23	*28*	*33*	*38*	*43*	*48*	*53*	*58*	*63*	*68*	*73*	*78*	*83*	*88*	*93*	*98*	Totals
*0**	18	7	1	2	7	2	10	3	15	7	5	6	3	2	1	1	90
1	10	10	8	3	7	5	3	5	4	3	1	4	3				66
2	4	7	4	3	4	4	2	2	2	6	3	2	1				44
3	1			1	2	4		2	1			1					12
4	3	2		2	1					2	1						11
5	3	4	3		1	1			1	2	1		1	1			18
6	3	2	2	4	3	1	1	3			2	1					22
7	4	2	1	1					2	2							12
8	1	1			2		1				1	1					7
9				1							1						2
10	2				1		2	1	2	1		1					10
11	1				1		2										4
12	1			1	1							1					4
13	1				1		2	1		2	1	1		2			11
14	2				2			1		1		1					7
15	1		1	1				1		2	1						7
16	2		1		1	1	1	1		3		1	1				12
17		1	2	1	1			2	1								8
18	2	2	1	1				2		3		1	1				13
19	1	1				1											3
20	1	2	4	1						1	1	1					11
Totals	61	41	28	22	35	19	24	24	28	35	18	22	10	5	1	1	374

* Centred at ·25 years.

TABLE XXII.

Adult Sister and Minor Brother.

Duration of Life of Adult Sister.

Duration of Life of Minor Brother.

	23	*28*	*33*	*38*	*43*	*48*	*53*	*58*	*63*	*68*	*73*	*78*	*83*	*88*	*93*	*98*	Totals
*0**	24	13	11	8	11	7	9	1	5	10	9	10	6	4			128
1	8	14	8	10	9	2	3	3	7	1	5	2	2		1		75
2	6	4	5	5	2	1		1	2	2	3	4	3				38
3	3	3	1	3		1				1	4	3					19
4	6	5	2	2		2	2			3	2	1	2	1			28
5	2	1		2		1											6
6	6	3					1	1	1								12
7	1	2	1	2		1	1		2			3					13
8	4	3	2	2	1	2							1				15
9			1	2	1	1		1			1						7
10		1	1														2
11	1	1															2
12																	0
13	2	2		1	1				1			1					8
14	1	1		1							1						4
15	2		1	2				1	1	3		1	2				13
16	1								1	1	1	1					5
17	3	4		2		2					1	1	1				14
18	2	1	4	1		1	1	1					3				14
19	2	2				1	1			2	1		1				10
20	2	1		1		2		1	1								8
Totals	76	61	37	44	25	24	18	10	21	23	28	27	21	5	1	0	421

* Centred at ·25 years.

VARIATION IN AURELIA AURITA.

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In 1895 an article on the Variation of the Tentaculocysts of *Aurelia aurita* (Linn.), written by me, was published in the *Quarterly Journal of Microscopical Science*. The chief object of the research, suggested by Professor Weldon, was the finding out the variation in the number of tentaculocysts, and if a variation occurred among the Ephyrae to see how far it affected the adults. The material for that investigation was collected in Plymouth Sound by the officials of the Marine Biological Association in 1893 and 1894. The results showed that there was a considerable variation in the number of tentaculocysts. The number of tentaculocysts (marginal sense-organs) is definitely fixed in the larval stage, commonly known as an Ephyra. A normal Ephyra has eight marginal lobes, or arms, each with a terminal sense-organ (tentaculocyst), and a prolongation of the stomach, forming a canal, runs along the middle of each lobe to the sense-organ.

A variation in the number of lobes affects also the number of tentaculocysts and the number of radial canals. The three are distinctly correlated, except when a twinning of the tentaculocysts occurs. This is very rare and only occurred five times in 1000 adult specimens.

It is easy to detect cases of twinning in the adults but not in the larval Ephyrae. Occasionally an Ephyra is found having two tentaculocysts at the end of an arm and the canal bifurcated. In some specimens, no doubt, this is genuine twinning, but it is possible for the two tentaculocysts to become widely separated by the growth of the umbrella, and each to have its own canal system. I have, therefore, included twinning in the variation as it does not practically interfere with the results.

It may be taken for granted, that, whatever number of tentaculocysts an Ephyra starts life with, that number remains constant during the whole life of that individual and that no further increase in number takes place during development.

The specimens collected in 1893 belonged to the Ephyra stage and the examination of 359 individuals showed that 22·6 p.c. had either more or less than the normal number (eight) of tentaculocysts. (Table I.)

TABLE I.

Percentage Comparison of the Numerical Variation in the Tentaculocysts of the specimens taken in 1893, 1894, *and* 1898.

		1893	1894	1894	1898	1898	1898
		Ephyrae	Ephyrae	Adults	Series A Small Adults	Series B Large Adults	A and B combined
		359	*1116*	*383*	*2000*	*1000*	*3000*
Number of Tentaculocysts	*4*				0·05	0·2	0·1
	5		0·1		0·1	0·1	0·1
	6	1·1	0·5	0·5	0·6	0·8	0·7
	7	2·2	3·0	4·7	3·7	4·7	4·1
	8	77·4	79·1	77·3	79·8	77·1	78·9
	9	6·1	6·7	8·6	5·7	7·5	6·3
	10	5·0	5·5	4·2	4·5	5·2	4·8
	11	3·3	3·1	2·6	3·6	1·9	3·0
	12	3·9	1·5	1·8	1·2	1·8	1·4
	13	0·8	0·3		0·3	0·5	0·4
	14		0·1		0·1	0·1	0·1
	15			0·3	0·1	0·1	0·1

In the spring of 1894 a larger number of Ephyrae were taken and 1116 specimens examined. The tentaculocysts were found to vary in 20·9 p.c.

In the summer of 1894 large adult specimens were caught and in 383 individuals it was found that 22·8 p.c. showed variation in the number of tentaculocysts.

Race of Aurelia in the River Tamar.

During my visit to the Marine Laboratory at Plymouth in 1898 I was glad to hear from my friend Mr E. W. L. Holt that he had seen Aurelia in shoals at Saltash. This town is situated on the River Tamar and is about three miles from Plymouth Sound, into which the river flows. It was near the mouth of this river that the specimens were taken in 1893 and 1894, so it is very probable that they also belong to the Tamar race of Aurelia. The river is tidal for about 20 miles and its water is very muddy and polluted. How far above and below Saltash the Aurelia ascend and descend with the tide has not yet been definitely ascertained, but under ordinary conditions they do not come down with the tide into Plymouth Sound. The Aurelia were first discovered by Mr Holt at Saltash in 1897, and since then they have been seen yearly in the Tamar. I think that the Tamar Aurelia may be safely regarded as a local race, and that they are born in the waters of the Tamar and spend their whole lives in the river or its tributaries.

In Plymouth Sound Aurelia is not at all a common jelly-fish, and shoals are rarely seen.

2000 *Small Adults taken in April*, 1898. *The Range and Numerical Variation of their Tentaculocysts.* (*Series A.*)

On April 23rd, Mr Holt and I started operations to catch Aurelia and tried first Barnpool, a small bay at the mouth of the Tamar, but we only caught two specimens, one at the surface and another in a shrimp-trawl. We then proceeded up the Tamar and put out the trawl net near the mouth of the river Lynher, one of the tributaries of the Tamar, about half a mile below Saltash. With two hauls of the trawl nearly 400 specimens were taken. The muddy state of the water prevented an Aurelia from being seen at a depth of about two feet below the surface, and as a very few were visible the hand-nets were not often used. In the evening of the same day Mr Holt saw a shoal of Aurelia swimming with the tide, close to the surface, near the Railway Bridge at Saltash. The next evening in that locality we caught with hand-nets over 1000 specimens. On April 27th, a second visit was made and we captured about 800 specimens.

The specimens collected on these occasions ranged from large Ephyrae up to small immature adults not exceeding 40 mm. in diameter. Very few Ephyrae were present as it was about a month too late for them.

I examined 2000 specimens and measured the diameter of the umbrella. All the measurements were made after preservation and the umbrella was perfectly flat, resting on the bottom of an ordinary photographic dish.

The numerical variation of the tentaculocysts is shown in Table II. The range of variation is from 4 to 15 tentaculocysts, and 20·2 p.c. of specimens have either more or less than the normal number.

TABLE II.

The Numerical Variation of the Tentaculocysts of 2000 *Adults, collected in April*, 1898.

Number of Tentaculocysts	Diameter of the Umbrella in Millimetres																				Total	Per-centage
Up to	*5*	*10*	*15*	*20*	*25*	*30*	*35*	*40*	*45*	*50*	*55*	*60*	*65*	*70*	*75*	*80*	*85*	*90*	*95*	*100*		
4			1																		1	0·05
5		2																			2	0·1
6		3	2	4	1	2															12	0·6
7		13	20	29	8	5															75	3·75
8		270	442	446	280	102	29	7	(normal)												1596	79·8
9		17	30	38	17	8	4	1													115	5·75
10		15	28	23	18	6	1														91	4·55
11		17	19	17	12	5	1	1													72	3·6
12		3	6	4	8	2	2														25	1·25
13		3	2	1	1																7	0·35
14		1				1															2	0·1
15				1	1																2	0·1
Totals		344	550	583	346	131	37	9													2000	100 p.c.

1000 *Large Adults taken in June*, 1898. *The Range and Numerical Variation of their Tentaculocysts.* (*Series B.*)

About a month later, on May 27th, and June 2nd, we again visited Saltash for the purpose of collecting a supply of large adult specimens, sufficient to ensure the possession of 1000 specimens in good condition. We tried, at first, to catch them with a large trawl-net, which was kept expanded by the tide from the Laboratory steamboat "Busy Bee" at anchor. This method had to be abandoned as the Aurelia got crushed in the net and most of them were found badly torn and damaged. To obtain specimens in perfect condition we found that catching them, one by one, in a hand-net, as they floated past the steamer, was the best method. The specimens were taken within two feet of the surface; the river being too muddy to see anything at a greater depth. I feel much indebted to my friends, Mr E. W. L. Holt, Mr W. I. Beaumont, and Mr L. W. Byrne, for their kindness in catching nearly all the specimens; my share of the work was the preservation.

This series of specimens ranged from 20 mm. to 95 mm. in diameter, and most of the larger ones carried planulae in the pouches on the oral arms, showing that they had reached full maturity. The majority of the specimens were from 40 mm. to 65 mm. in diameter.

The numerical variation of the tentaculocysts is given in Table III. The range of variation is the same as that of the small adults collected in April, but the percentage of specimens showing a variation is greater, 22·9 p.c., being 2·7 p.c. more than in the small adults.

TABLE III.

The Numerical Variation of the Tentaculocysts of 1000 *Adults, collected in June*, 1898.

		Diameter of the Umbrella in Millimetres																			Total	Per-centage
	Up to	*5*	*10*	*15*	*20*	*25*	*30*	*35*	*40*	*45*	*50*	*55*	*60*	*65*	*70*	*75*	*80*	*85*	*90*	*95*	*100*		
Number of Tentaculocysts	*4*									1	1											2	0·2
	5										1											1	0·1
	6									1	3	1	1	1	1							8	0·8
	7						1	1	3	11	10	2	4	5	3	5	1	1				47	4·7
	8	(normal)				4	11	27	39	97	128	138	111	83	57	37	22	9	7	1		771	77·1
	9					1	3	2	4	13	12	12	8	4	6	6	1		2	1		75	7·5
	10								3	7	7	14	9	7	4		1					52	5·2
	11				1			1	1	1	4	2	3	5	1							19	1·9
	12						1		2	5	1	3	1	2	1	2						18	1·8
	13							2	1	1			1									5	0·5
	14												1									1	0·1
	15										1											1	0·1
Totals					1	5	16	33	53	137	168	172	139	107	73	50	25	10	9	2		1000	100 p.c.

The Combination of the Small and Large Adults.

The result of the combination of the small adults (Table II.) and the large adults (Table III.) is shown in Table IV. which is illustrated by a curve (1) showing the frequency of the tentaculocysts.

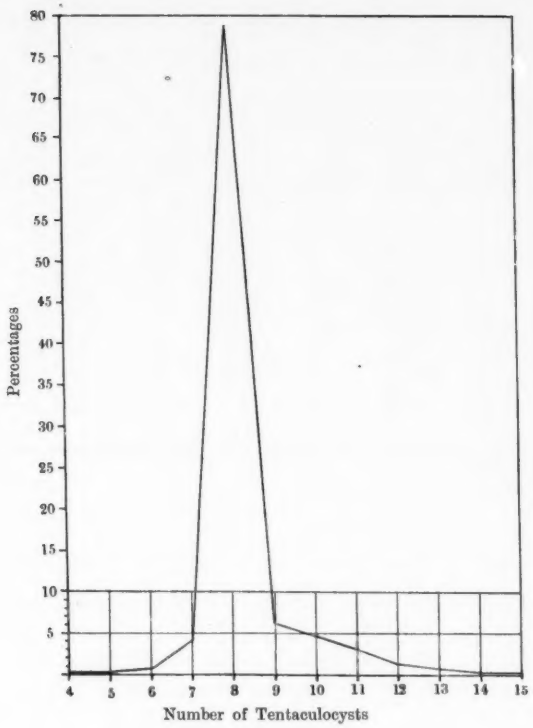
This combination gives a grand total of 3000 specimens belonging to the same race, and it shows that 21·1 p.c. have more or less than the normal number of tentaculocysts. I think that 21 p.c. may be regarded as the normal amount of variability of the Tamar race.

The table also shows that 4·9 p.c. of the specimens have less (4 to 7) than the normal number of tentaculocysts, and that 16·1 p.c. have an excess (9 to 15) of the normal number.

TABLE IV.

The Numerical Variation of the Tentaculocysts of 3000 *Adults.*
(*Tables II. and III. combined.*)

		Diameter of the Umbrella in Millimetres																			Total	Per-centage
	Up to	*5*	*10*	*15*	*20*	*25*	*30*	*35*	*40*	*45*	*50*	*55*	*60*	*65*	*70*	*75*	*80*	*85*	*90*	*95*	*100*		
Number of Tentaculocysts	*4*			1						1	1											3	0·1
	5		2								1											3	0·1
	6		3	2	4	1	2			1	3	1	1	1	1							20	0·67
	7		13	20	29	8	6	1	3	11	10	2	4	5	3	5	1	1				122	4·07
	8		270	442	466	284	113	56	46	97	128	138	111	83	57	37	22	9	7	1		2367	78·9
	9		17	30	38	18	11	6	5	13	12	12	8	4	6	6	1		2	1		190	6·33
	10		15	28	23	18	6	1	3	7	7	14	9	7	4		1					143	4·77
	11		17	19	18	12	5	2	2	1	4	2	3	5	1							91	3·03
	12		3	6	4	8	3	2	2	5	1	3	1	2	1	2						43	1·43
	13		3	2	1	1		2	1	1			1									12	0·4
	14		1				1						1									3	0·1
	15				1	1					1											3	0·1
Totals			344	550	584	351	147	70	62	137	168	172	139	107	73	50	25	10	9	2		3000	100



CURVE 1.

Percentage curve showing the numerical variation in the tentaculocysts of 3000 adults taken in 1898. (Table IV.)

Comparison between the Smallest and Largest Adults.

The measurement of the diameter of the umbrella of all the specimens was made in order that the variability of the smallest immature adults might be compared with that of the large mature ones.

The specimens have been separated into four groups for comparison (Table V.). The percentage of variability is practically the same in the smallest (5 to 10 mm.) and the largest (56—95 mm.) adults. Curve 2. The percentage of specimens with less than the normal number of tentaculocysts is almost the same in both groups and it therefore follows that that with the excess is also the same.

It appears, then, that neither a decrease nor an increase in the number of tentaculocysts is injurious to the race, and that no selection is taking place in the young and adult stages in the number of tentaculocysts.

TABLE V.

Comparison between the Smallest and Largest Adults taken in 1898.

		5 to 15 mm. Umbrella-diameter		46 to 55 mm. Umbrella-diameter		5 to 10 mm. Umbrella-diameter		56 to 95 mm. Umbrella-diameter	
		Number of Specimens	Per-centage	Number of Specimens	Per-centage	Number of Specimens	Per-centage	Number of Specimens	Per-centage
Number of Tentaculocysts	*4*	1	0·11	1	0·29	0		0	
	5	2	0·22	1	0·29	2	0·58	0	
	6	5	0·56	4	1·18	3	0·87	3	0·72
	7	33	3·69	12	3·53	13	3·78	19	4·58
	8	712	79·65	266	78·24	270	78·49	327	78·79
	9	47	5·26	24	7·06	17	4·94	28	6·75
	10	43	4·81	21	6·18	15	4·36	21	5·06
	11	36	4·03	6	1·76	17	4·94	9	2·17
	12	9	1·00	4	1·18	3	0·87	6	1·45
	13	5	0·56	0		3	0·87	1	0·24
	14	1	0·11	0		1	0·29	1	0·24
	15	0		1	0·29	0		0	
Totals		894	100	340	100	344	99·99	415	100

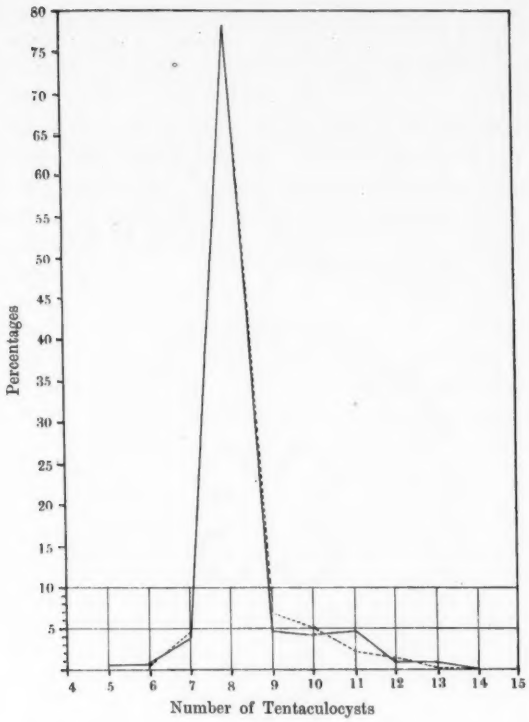
Combination of the Ephyrae and Adults taken in 1893, 1894, *and* 1898.

As evidence is in favour of the specimens taken in 1893 and 1894 belonging to the Tamar race of Aurelia, a Table (VI.) is given showing the results of combining all the specimens into one series, giving a total of 4858 specimens. These show that 21·29 p.c. have more or less than the normal number of tentaculocysts and the individual percentages compared with those of the 3000 taken in 1898 (Table III.) show a great similarity.

TABLE VI.

The Numerical Variation in the Tentaculocysts of the specimens taken in 1893, 1894, *and* 1898.

Year	Stage	Number of Specimens	Number of Tentaculocysts											
			4	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*
1893	Ephyra	359	0	0	4	8	278	22	18	12	14	3	0	0
1894	Ephyra	1116	0	1	6	34	883	75	61	35	17	3	1	0
1894	Adult	383	0	0	2	18	296	33	16	10	7	0	0	1
1898	Adult	3000	3	3	20	122	2367	190	143	91	43	12	3	3
Totals		4858	3	4	32	182	3824	320	238	148	81	18	4	4
Percentages			0·06	0·08	0·66	3·75	78·72	6·59	4·90	3·05	1·67	0·37	0·08	0·08



CURVE 2.

Percentage curves of the smallest adults (5 to 10 mm. in diameter) and the largest adults (56—95 mm.) taken in 1898. (Table V.)

Continuous line for the smallest adults.

Dotted line for the largest adults.

Further Observations on the 1000 *Large Adults taken in June. (Series B.)*
A Correlation between the Radial Canals and the Tentaculocysts.

The excellent condition of the large adults afforded a good opportunity of examining the radial canal system. The few figures given by Romanes and other observers show distinctly that there is a correlation between the radial canals and the tentaculocysts, and my observations confirm theirs.

In the 1000 adult specimens I found that every tentaculocyst (twinning excepted) had its own branched canal leading direct from the stomach, and always separated from the adjacent branched canals by a simple unbranched canal. By means of the canals the origin of the tentaculocyst could be traced home to either the perradial or interradial system.

An increase or decrease in the number of tentaculocysts usually leads to an asymmetrical arrangement of the canals, so that a tentaculocyst situated in a strictly adradial position may be traced by its canal to either the perradial or interradial system.

Three diagrams have been prepared to illustrate the correlation of the radial canal system and the tentaculocysts. Diagram I. shows the arrangement of the canals and tentaculocysts in a normal specimen. There are four branched perradial canals and four branched interradial canals which are separated by simple unbranched adradial canals. The branched canals are all of the same pattern, but show a considerable amount of variation.

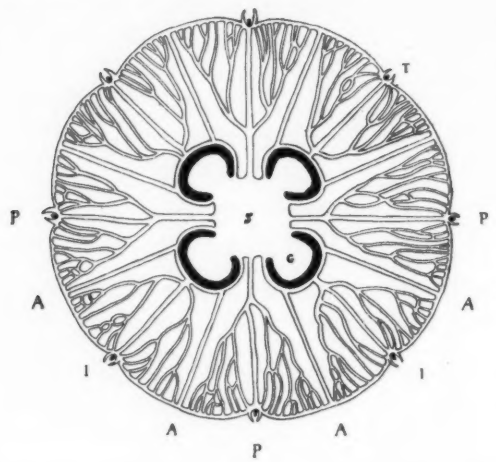


DIAGRAM I.

The normal arrangement of the radial canal system in a normal specimen.

P, perradial. *I*, interradial. *A*, adradial. *G*, genital sac with gonad. *S*, stomach. *T*, tentaculocyst.

In the diagrams the oral arms are omitted (they are perradially situated) and also the tentacles which form a fringe round the margin of the umbrella.

The normal type is seen at the top of Diagram I. (perradial). There is a straight canal running direct from the stomach to the tentaculocyst, and it gives off two opposite branches, each of which is subdivided into many branches, all entering the ring canal. There are many modifications of this type and several are illustrated in the diagrams. The lateral branches are often not opposite each other, and in some specimens the main canal looks like a branch from one of the lateral canals.

The interradial branched canal is of the same pattern as the perradial. In some specimens it looks however quite different, but it is only a modification of

the normal type and probably due in many cases to the outward growth of the genital sacs cutting off the base of the canal. Instead of one canal leaving the stomach there are three. The central one runs direct from the stomach to the tentaculocyst and the lateral ones are branched.

The adradial canals are always simple and unbranched and never run to a tentaculocyst.

Specimens which have less than the normal number of tentaculocysts have usually a corresponding decrease in the number of branched radial canals. If one perradial tentaculocyst be absent, the branched canal is also absent and its place taken by a simple unbranched canal. The loss of one or more branched canals does not produce a vacant space, the adjacent branched canals spread out and occupy a larger area.

A decrease in the number of tentaculocysts may occur through injury done to the umbrella. This can, as a rule, be detected by a careful examination of the margin with a lens. The presence of a branched canal without a tentaculocyst is suspicious, and if there were any traces of an old injury which had healed up, leaving a slight or an irregular margin to the umbrella, the specimen was rejected as a damaged one.

In a few specimens a branched canal without a tentaculocyst was found with the margin in a perfect condition and these were considered cases of genuine variation.

Anastomosing of the radial canals by cross branches forming a kind of network occurs in nearly all the large adults. It is rarely present in the small adults. The amount of anastomosing is very variable and not always regularly distributed amongst the radial canals. Often one or two quadrants in a specimen show considerable anastomosing, whilst the other quadrants are comparatively free from it. In two specimens, having the same sized umbrella, one may show a considerable amount of anastomosing, the other practically none at all. The adradial canals are often anastomosed to the branches of the perradial and interradial canals.

The Position of the Tentaculocysts in Specimens having a Decrease in Number.

In my previous publication on Aurelia there is a sentence which is rather misleading and requires an explanation. "An examination of the specimens (adults) does not show that any particular position on the margin of the umbrella is favoured either by an increase or decrease of the tentaculocysts." If the position of the tentaculocysts on the margin of the umbrella were marked down on skeleton diagrams, no attention being paid to the radial canal system, then the above statement would be correct. The material upon which that statement was based (adults of 1894) was not in first class condition and no attempt was made to

examine the radial canal system, but only the number of tentaculocysts was marked on skeleton diagrams.

In the 1000 adults (Series B) there are 58 specimens (5·8 p.c.) with less than the normal number of tentaculocysts and the majority show that the absent tentaculocyst belongs to the perradial system.

The exact position of the absent tentaculocysts cannot be conveniently shown by the use of tables, but full details are given below. Specimens with less than the normal number of genital sacs have usually a decrease in the number of radial canals and tentaculocysts (Table VII.).

The Position of the Tentaculocysts in Specimens having an Increase in Number.

No less than 171 specimens (17·1 p.c.) show an increase in the number of tentaculocysts compared with 5·8 p.c. showing a decrease. This excess is one of the characteristic features of the variation and occurs in every series examined (Tables I. and VI.) and it clearly shows that the tendency is towards an increased number of sense-organs. It was to find out whether the perradial or the interradial series were more subject to variation that the radial canal system was examined, as it was the only reliable means of exactly locating the original source of the tentaculocysts. A tentaculocyst may be adradially, or nearly adradially, situated on the margin of the umbrella, but its canal may curve and join the perradial system or it may curve the opposite way and belong to the interradial system.

I have not called any tentaculocysts or branched canals adradial, though they may be in some specimens adradially situated. The increase in the perradial system is simply a case of doubling or trebling the number of canals and tentaculocysts; in the interradial system only doubling occurs. To call such canals and tentaculocysts adradial would imply that the interradial canal and tentaculocyst were absent, and this is not the case (Diagram III.).

The examination of the canal system shows that the increase is generally perradial and details are given on page 102.

Specimens with more than the normal number of genital sacs have usually an increase in the number of radial canals and tentaculocysts (Table VII.).

If a very slow and gradual change is taking place in the number of tentaculocysts then the tendency is towards the establishment of a race with ten tentaculocysts, due to an increase of two opposite perradial tentaculocysts arranged thus:—

Perradial	2.1.2.1.
Interradial	1.1.1.1.

If no check occurs and the variation still continues in the perradial system the ultimate result will probably be a race with twelve tentaculocysts arranged thus:—

Perradial 2.2.2.2,
Interradial 1.1.1.1,

the medusa having eight perradial tentaculocysts and canals, four interradial tentaculocysts and canals, and eight adradial unbranched canals.

A Detailed Account, giving the Position of the Tentaculocysts in the Specimens (Large Adults belonging to Series B) having more or less than the Normal Number.

Four Tentaculocysts. (*Two specimens.*)

Perradial 0.0.0.0.
Interradial 1.1.1.1.

One specimen with only four interradial tentaculocysts.

The interradial branched canals occupy the four quadrants. There are no adradial canals, but four simple unbranched canals, perradially situated. It is probable that in this specimen the perradial canal system is absent and the simple canals, perradially situated, are the representatives of the adradial system removed from their proper position.

Perradial 1.1.1.
Interradial 0.0.1.

One specimen with three genital sacs and three oral arms.

Five Tentaculocysts. (*One specimen.*)

Perradial 1.0.1.
Interradial 1.1.1.

This specimen has three genital sacs and three oral arms.

Six Tentaculocysts. (*Eight specimens.*)

Perradial 1.0.1.0. 1.1.0.0.
Interradial 1.1.1.1. 1.1.1.1.

Two specimens with two perradial tentaculocysts absent.

Perradial 1.0.1.1.
Interradial 1.1.0.1.

Two specimens with one perradial and one interradial tentaculocyst absent.

Perradial 2.0.2.0.
Interradial 0.1.0.1.

The above specimen is bilaterally symmetrical. In each half of the umbrella there are three branched canals with tentaculocysts.

Perradial	1.1.1.
Interradial	1.1.1.

Three specimens with three genital sacs and three oral arms.

Seven Tentaculocysts. (*Forty-seven specimens.*)

Perradial	0.1.1.1.
Interradial	1.1.1.1.

Thirty specimens with one perradial tentaculocyst absent.

Perradial	1.1.1.1.
Interradial	0.1.1.1.

Fourteen specimens with one interradial tentaculocyst absent.

When a perradial or an interradial tentaculocyst is absent the branched radial canal is usually absent and in its place is a simple unbranched canal.

Perradial	1.1.1.	1.2.1.
Interradial	1.2.1.	1.1.1.

Three specimens with three genital sacs and three oral arms.

Nine Tentaculocysts. (*Seventy-five specimens.*)

Perradial	2.1.1.1.
Interradial	1.1.1.1.

Sixty-three specimens have the extra tentaculocyst perradially situated.

In the place of a single branched perradial canal there are two perradial branched canals usually occupying about the same space as a normal single branched canal.

In two specimens the increase of tentaculocysts is due to twinning.

There is one normal branched canal with two tentaculocysts very close together, just separated by a few marginal tentacles.

Perradial	2.0.2.1.
Interradial	1.1.1.1.

One specimen with two extra opposite perradial tentaculocysts and one perradial tentaculocyst and branched canal absent.

Perradial	2.1.1.1.	2.1.1.1.
Interradial	2.1.1.0.	2.1.0.1.

Three specimens with an extra perradial tentaculocyst, and an extra interradial in one quadrant, but one absent in the adjacent or opposite quadrant.

Perradial	1.1.1.1.
Interradial	2.1.1.1.

Five specimens have an extra interradial tentaculocyst.

In one specimen the increase is due to twinning.

Three specimens with more than the normal number of genital sacs.

One with six genital sacs and six oral arms.

One with five genital sacs and five oral arms.

One with three genital sacs and three oral arms, and the tentaculocysts arranged thus:—

Perradial	3.2.2.
Interradial	0.2.0.

Ten Tentaculocysts. (*Fifty-two specimens.*)

Perradial	2.2.1.1.
Interradial	1.1.1.1.

Nineteen specimens have an extra tentaculocyst in adjacent quadrants.

Perradial	2.1.2.1.
Interradial	1.1.1.1.

Seventeen specimens have an extra tentaculocyst in opposite perradii. In one specimen the increase is due to twinning which occurs in the opposite perradii.

Perradial	3.1.1.1.
Interradial	1.1.1.1.

Three specimens have two extra perradial tentaculocysts in the same perradius.

Perradial	2.1.1.1.
Interradial	1.1.2.1.

Six specimens with an extra perradial and an extra interradial tentaculocyst.

Seven specimens have more than the normal number of genital sacs and oral arms.

Five with six genital sacs and six oral arms. Radial canals and tentaculocysts arranged thus:—

Perradial	1.0.1.0.1.1.	1.0.1.1.1.1.
Interradial	1.1.1.1.1.1.	1.1.1.0.1.1.

Two specimens with five genital sacs and five oral arms. Radial canals and tentaculocysts arranged thus:—

Perradial	1.1.1.1.1.
Interradial	1.1.1.1.1.

Eleven Tentaculocysts. (*Nineteen specimens.*)

Perradial	2.2.2.1.
Interradial	1.1.1.1.

Eleven specimens have three extra perradial tentaculocysts. In one of the specimens the increase of one tentaculocyst is due to twinning.

Perradial	2.1.2.1.	2.2.1.1.
Interradial	1.2.1.1.	1.2.1.1.

Two specimens have two extra perradial tentaculocysts and one extra interradial.

Perradial	1.2.1.1.
Interradial	1.2.1.2.

One specimen with one extra perradial and two extra interradial tentaculocysts.

Perradial	1.1.1.1.
Interradial	2.2.2.1.

One specimen with the normal number of perradial tentaculocysts, but with three extra adjacent interradial tentaculocysts.

Five specimens with more than the normal number of genital sacs.

One with ten genital sacs and ten oral arms.

Three with six genital sacs and six oral arms.

One specimen with five genital sacs and five oral arms.

Twelve Tentaculocysts. (*Eighteen specimens.*)

Perradial	2.2.2.2.
Interradial	1.1.1.1.

Six specimens have an extra tentaculocyst in each of the four perradii.

Perradial	3.2.1.2.	3.2.2.1.	3.3.1.1.
Interradial	1.1.1.1.	1.1.1.1.	1.1.1.1.

Three specimens have extra tentaculocysts all perradially situated, but only in two or three perradii as shown in the above formulae.

Perradial	2.2.2.1.	2.2.1.1.
Interradial	2.1.1.1.	1.2.1.2.

Three specimens have extra perradial and interradial tentaculocysts as shown in the above formulae.

Six specimens have more than the normal number of genital sacs.

Four with six genital sacs and six oral arms. Tentaculocysts arranged thus:—

Perradial	1.1.1.1.1.1.
Interradial	1.1.1.1.1.1.

Two with five genital sacs and five oral arms. Tentaculocysts arranged thus:—

Perradial	1.1.1.1.1.
Interradial	1.2.1.2.1.

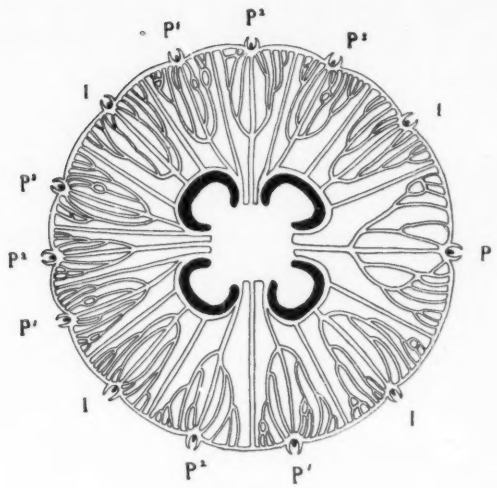


DIAGRAM II.

The arrangement of the radial canals in a specimen with thirteen tentaculocysts.

Perradial 3.3.1.2.
Interradial 1.1.1.1.

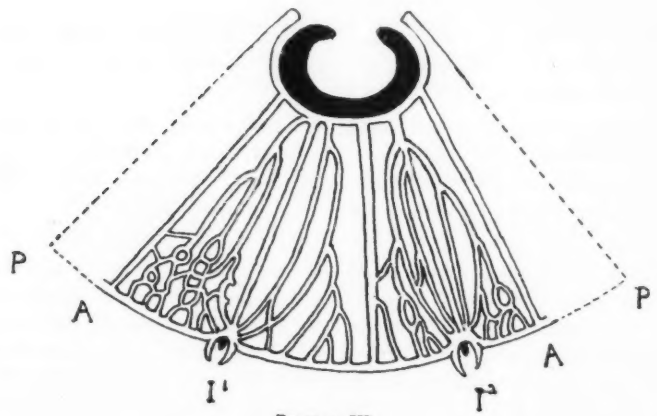


DIAGRAM III.

The arrangement of the interradial canals when an increase occurs in the number of interradial tentaculocysts.

Thirteen Tentaculocysts. (*Five specimens.*)

Perradial	3.2.2.2.	3.3.1.2.
Interradial	1.1.1.1.	1.1.1.1. (Diagram II.)

Two specimens with five extra perradial tentaculocysts.

Perradial	2.2.2.2.
Interradial	2.1.1.1.

One specimen with four extra perradial tentaculocysts and one extra interradial.

Two specimens with more than the normal number of genital sacs.

One with six genital sacs and six oral arms.

One with five genital sacs and five oral arms.

Fourteen Tentaculocysts. (*One specimen.*)

This specimen has eight genital sacs and eight oral arms.

Fifteen Tentaculocysts. (*One specimen.*)

Perradial	2.1.3.3.
Interradial	1.2.1.2.

There are five extra perradial and two extra interradial tentaculocysts.

Numerical Variation of the Genital Sacs of 3000 *Adults taken in* 1898. (*Series A and B.*)

Previous accounts by other writers on the Variation of Aurelia have been mostly confined to the variation of the genital sacs, and numerous illustrations have been published. (Ballowitz, 1899.)

In the Ephyra stage the genital sacs are not present, but their position and number are marked by the bundles of gastric filaments. Normally there are four interradial bundles of filaments. The gastric filaments are not figured in the diagrams of the adults. They are always situated along the inner side of the gonads.

In the small Ephyrae an increase or decrease in the number of gastric filaments indicates that the specimen will probably have an increase or decrease in the number of genital sacs, as the numbers of bundles and gonads are correlated. It is not however easy to count the number of bundles of gastric filaments in the small Ephyrae, especially when there are indications of an increase, as then owing to the closeness of the bundles they form occasionally an unbroken ring. When an Ephyra has developed into the adult form and the gonads begin to appear, and also the genital sacs, then any variation in the number of gonads or genital sacs is easily seen.

The 3000 specimens taken in 1898 show that 2·4 p.c. have more or less than the normal number (four) of genital sacs. The range of variation is from 2 to 10, and most of the specimens show an increase in number (Table VII.).

TABLE VII.

The Numerical Variation of the Genital Sacs of 3000 *specimens and its relation with the Numerical Variation of the Tentaculocysts.*

		Number of Genital Sacs									Abnormal Cases
		2	*3*	*4* Normal	*5*	*6*	*7*	*8*	*9*	*10*	
Number of Tentaculocysts	*4*	1	1	(*1*)							2
	5		2	(*1*)							2
	6		6	(*14*)							6
	7		5	(*117*)							5
	8		2	(*2363*)	1			1			4
	9		1	(*185*)	2	2					5
	10			(*131*)	4	8					12
	11			(*73*)	3	10	4			1	18
	12			(*30*)	3	10					13
	13			(*10*)	1	1					2
	14			(*2*)				1			1
	15			(*1*)			2				2
Totals		1	17	(*2928*)	14	31	6	2	0	1	72

There is a perfect correlation between the number of genital sacs and the number of oral arms, and I have not found any exceptions; a specimen with six genital sacs has always six oral arms.

There is not a perfect correlation between the number of genital sacs and the number of radial canals and tentaculocysts, though a decrease in the number of genital sacs is usually accompanied by a decrease in the number of radial canals and tentaculocysts, and an increase by an increase in the number of the radial canals and tentaculocysts.

In the 3000 adults 21·1 p.c. show a variation in the number of tentaculocysts and 2·2 p.c. of these have an abnormal number of genital sacs. But the variation in the number of genital sacs has a considerable effect upon the numerical variation of the tentaculocysts. Only four specimens out of 72 with an abnormal number of genital sacs have eight tentaculocysts.

The adults collected in 1894 (383 specimens) showed 2·08 p.c. with an abnormal number of genital sacs.

Method used for the Preservation of Aurelia.

The large adults were placed in buckets containing Formaline 5 p.c. solution in sea-water. (Formaldehyde 5 c.c. and sea-water 95 c.c.) They quickly died in a good state of expansion. When the bucket was full of specimens about 100 c.c. of strong Formaldehyde was added. The specimens were allowed to soak in this for six hours and occasionally stirred up. After this preliminary soaking in Formaline they were removed, one by one, and placed in a mixture of Formaline and Chromic Acid.

Formaline, 10 p.c. in freshwater .....9 volumes.
Chromic Acid, 5 p.c. solution .....1 volume.

They were left in this mixture for 24 hours; occasionally stirred up and a little strong Formaldehyde added. It is very important that the specimens should be thoroughly saturated with Formaline. The specimens were removed from the Chromic-Formaline solution and finally stored in bottles containing Formaline 10 p.c. in freshwater. I prefer storing in a 10 p.c. solution of Formaline, as there is then a fair margin against deterioration of strength.

These specimens are now three years old and are still in excellent condition. The addition of Chromic Acid greatly improves the specimens. The opaque whiteness, produced when Formaline only is used, is not so conspicuous and the jelly becomes more translucent. The specimens are also tougher and more pliable and can be handled without fear of damage. The yellowish-brown colour due to the Chromic Acid gradually disappears owing to oxidation and the specimens turn to a pale bluish-green. An ordinary black xylonite photographic dish is an excellent thing to examine the specimens in. The black background shows up the radial canal system.

References.

Ballowitz, E., 1899. "Ueber Hypomerie und Hypermerie bei Aurelia aurita." *Archiv für Entwickelungsmechanik der Organismen*, Bd. VIII. pp. 239—252.

(This paper contains a good Bibliography and summary of the literature relating to the variation of Aurelia.)

Browne, E. T., 1895. "On the Variation of the Tentaculocysts of Aurelia aurita." *Quart. Journ. Micro. Sci.*, Vol. XXXVII. pp. 245—251.

A FIRST STUDY OF NATURAL SELECTION IN CLAUSILIA LAMINATA (MONTAGU).

By W. F. R. WELDON.

Among the terrestrial mollusca of Europe are many species which have certainly inhabited their present areas of distribution for a very long time*. The shells of some species are and have been exceedingly variable, while others are, and have apparently for long periods remained, almost invariable.

The problems presented by species belonging to either category are of great interest in connection with the theory of natural selection; and the characters which can be studied in recent and in fossil shells alike are often well fitted for numerical treatment. Such characters as the shape of the spire, the number of ridges and furrows on a given whorl, the size and shape of the aperture, can often be expressed in such a way as to admit of numerical comparison between individuals; these are the characters used in the discrimination of "species"; and in the present state of our knowledge they afford excellent examples of specific characters which appear to us "useless."

A typical example of a species which appears to have varied very little during a long period of time is *Clausilia laminata* (Montagu): and I wish now to record an attempt to determine the variability of one element of its spire.

If the shell of an adult *Clausilia laminata* be carefully ground upon a soft stone, it is possible to expose a section passing with sensible exactness through the plane which contains the axis of the spire. The columella is in many cases approximately straight, except near its lower end, where it bends abruptly and supports the clausilium. This bend in the columella is associated with a change in the character of the spiral, leading to the formation of the oblique aperture of the finished shell, but above the point where the bend occurs the columella may without serious error be treated as a conical tube, some ten or twelve millimetres long, and about half a millimetre in diameter at its lower (broader) end. If a shell be ground on a flat stone, until the middle of this long and narrow tube is

* For a discussion of the history of land shells in the Palaearctic Region, see Kobelt; *Studien zur Zoogeographie*, Wiesbaden 1897–98.

apparently reached through practically its whole length, the section exposed will be very near indeed to that containing the real axis of the spire. Such a section will have the appearance shown in Fig. 1; and from it we can ascertain the elements which determine the "pitch" and certain other characters of the spire.

The shell is of course essentially a tube, coiled round an axis so that the successive coils are in contact. The wall of the tube nearest to the axis forms the columella; that remote from the axis forms the outer surface of the shell. Where the successive whorls are in contact, a double septum is formed, running from the columella to the outer surface of the shell. The line of contact of successive whorls along the outside of the shell describes what we will call the "peripheral spiral"; the line of contact along the axial wall may be called the "columellar spiral*." If we know the pitch of these two spirals, and their distance apart at every point, we know the elements which determine all the important characters of the spire, except the "ventricosity," which is measured by the longitudinal curvature of the outer walls, for example by the convexity of the portion *BDE* in Fig. 1.

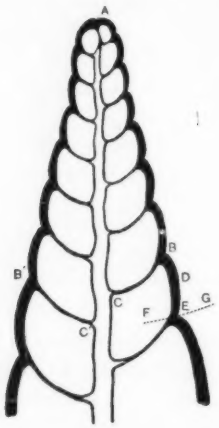


Fig. 1.

Now in a median longitudinal section through the shell, the relations of any point on the peripheral spiral, such as *B*, Fig. 1, to the corresponding point on the columellar spiral, *C*, and to the apex of the shell *A*, are fully determined if we know the lengths of the sides of the triangle formed by joining the three points *A*, *B* and *C*. If such a triangle be supposed to revolve round the columella, the length of each side changing as it revolves, while the angle *A* remains always

* I have deliberately used the word "spiral" to describe a curve in three-dimensional space, which mathematicians would call a "helix," in order to use a term which may be applied to pulmonates generally, without the inconvenience which would result from speaking for example of the "helix" of *Helix aspersa*.

at the apex of the shell, the angle B will describe a spiral which will be identical with the peripheral spiral of the shell if the law of change of the side AB is the same as that of the peripheral radius vector of the shell; and by finding an appropriate law of change for the sides AC and BC, the angle C may be made to describe the columellar spiral, while the side BC maintains the two spirals at their proper distance apart. We may therefore consider the two fundamental spirals, upon which the main characters of the shell depend, as generated by a triangle revolving round a long and narrow cone, so that one side always touches the cone, and one angle always lies at the apex of the cone, the length of each side changing as the triangle revolves.

In actual sections of shells, such as that drawn in Fig. 1, the peripheral and columellar spirals are each cut from 17 to 21 times; so that the elements of the fundamental triangle can be determined a great number of times. Further, since the section is approximately flat, the interval between any position ABC and the next position $AB'C'$ in which the triangle can be measured, corresponds to a revolution through 180°. From such a number of data it should be possible to determine the law of change of the triangle with considerable accuracy. The measurements required are somewhat laborious, and I have not at present determined the rate of change of every side of this fundamental triangle, but only that of the side AB, the radius vector of the peripheral spiral. Such a determination leaves the distance between the columellar and peripheral spirals uncertain: it only determines that the radius of the peripheral spiral at any point lies somewhere on the arc of such a circle as that shown at FEG, Fig. 1. The results of this determination seem to me of some interest, so that I venture to publish them before the other measures are completed. A reason for considering these measures separately from the others is that they do not seem to be affected by a curvature of the upper part of the columella, which is often sensible, and sometimes considerable. The shells measured were chosen at random, and include some in which the columella is very sensibly bent.

The chief material used was collected during the summer of last year and during the present summer in the great beech-wood known as Der Holm at Gremsmühlen on the Dieck See, a lake on the eastern border of Holstein. In this wood *Clausilia laminata* was extremely abundant, occurring together with *C. biplicata*. Other species also occurred, but none were so common as *C. laminata*. The conspicuous species of *Helix* were *H. lapicida*, *H. nemoralis*, *H. arbustorum*, *H. hispida* and *H. rotundata*, all of which were exceedingly common. The forest slopes down to the very edge of the lake, and the moist nature of the ground where the *Clausilia* were collected is shown by the frequent presence of species of *Succinea*. The soil is the light sandy alluvium so common throughout Holstein, and the whole wood is probably less than 150 feet above the sea level.

The peripheral radii were measured in sections of 100 adult shells from this wood. The measures were recorded to 0·01 millimetre, and I believe they are fairly trustworthy. They were made with an instrument designed for me by

Mr Horace Darwin, and constructed by the Cambridge Scientific Instrument Company, which consists essentially of a long and flat piece of plate glass capable of movement in a plane, parallel to itself, under two compound microscopes. The part of the glass plate which moves under one microscope carries a scale, divided into tenths of a millimetre, while the microscope over it has a vernier in the eye-piece; the part of the plate moving under the second microscope carries the object to be measured, the microscope over it having cross wires in the eye-piece. The object is adjusted so that the organ to be measured is parallel to the line of motion of the glass plate, and the scale is read when first one extremity of the organ measured, and then the other, lies under the intersection of the cross wires, the difference between the two readings of the scale giving the required length. The only serious source of error is a slight uncertainty which sometimes exists about the exact position of the apex; but this does not, I believe, involve an uncertainty of 0·01 mm. in any but the uppermost measures, which are unfortunately also the shortest. An inspection of Fig. 1 will show that a small uncertainty in the position of A (which can only affect its position in a direction perpendicular or nearly so to the axis of the shell) will not have an important effect upon the estimated length of any radius inclined at an angle of less than 45° to the columella. The error from obliquity in the plane of the sections measured is certainly too slight to affect measures involving only quantities of the order of 0·01 mm.

Before the measures of the radii of two shells can be compared, it is necessary to find some way of determining the plane in which the section of either shell cuts it. It is clear that an infinite number of planes can cut the shell so as to pass through the axis, and there is no obvious means of finding out which of these contains the actual section measured. There is therefore no means of knowing the angle through which the first radius measured in any section has revolved before meeting the plane of the section. Since the angular distance of the plane of the section from the origin of the spiral cannot be determined, it is necessary to choose some arbitrary plane to which each section may be referred; and after consultation with Professor Pearson, all sections were referred to a plane containing a radius of the columellar spiral 5 mm. in length.

The distance of such a plane from the plane of any actual section was found by interpolation between two adjacent columellar radii. For example in Fig. 1, if the columellar radius AC were exactly 5 mm. in length, the plane of the section would exactly coincide with the plane of reference; if however AC were (as in the first individual of the series) 4·68 mm. long, and the next columellar radius AC' were 5·27 mm. long, then the plane of the section would be inclined to the plane of reference at a certain angle. Assuming the columellar spiral to be sensibly equiangular through 180° (an assumption fairly in accord with observation) the angle between the plane of the section and the plane of reference may be taken to be

$$\frac{5-4{\cdot}68}{5{\cdot}27-4{\cdot}68}\times 180^\circ = 0{\cdot}5424\times 180^\circ;$$

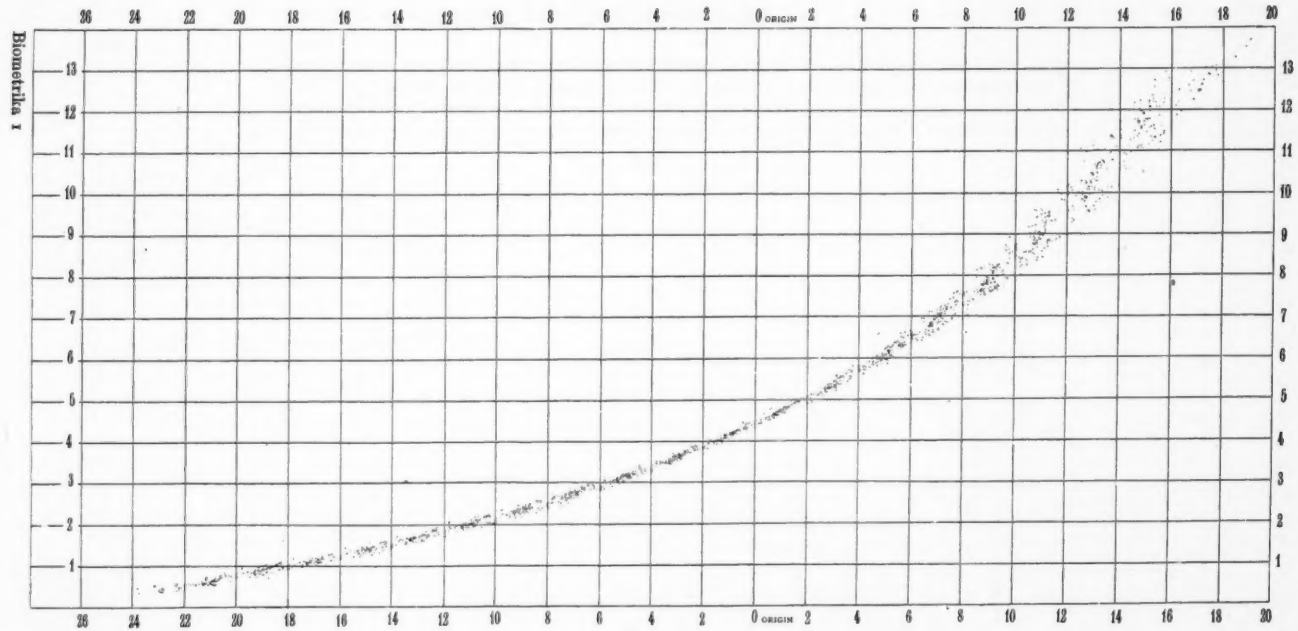


FIG. 2.

Diagram showing the length of every peripheral radius measured in 100 adult shells of *Clausilia laminata* from Gremsmühlen. The ordinate of each dot gives the length of a radius, the abscissa giving its angular distance from the plane of the standard columellar radius 5 mm. long. Each vertical unit of the scale denotes one millimetre of radial length; each horizontal division denotes two right angles of angular distance. Angular distances from the plane of the standard columellar length towards the apex of the shell are negative, those towards the mouth of the shell are positive.

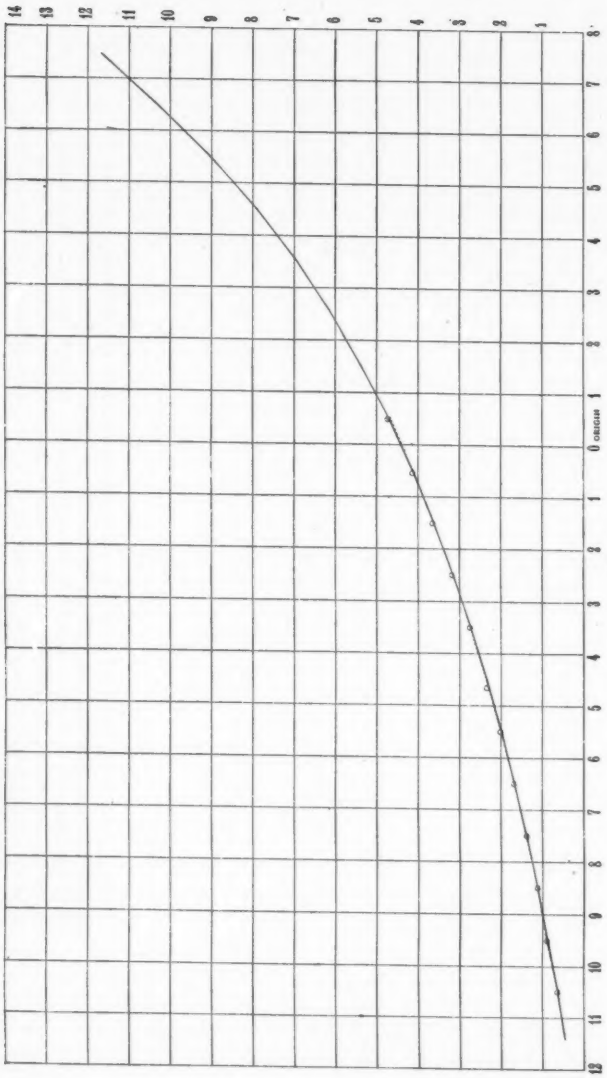


FIG. 3.

The continuous line shows the mean length of peripheral radii, at any angular distance from the standard columellar radius, in 100 adult shells from Gremsmühlen; the dotted line shows the mean length of corresponding radii in 100 half-grown shells from the same place. The dots surrounded by circles show the mean length of peripheral radii in the upper whorls of 32 adult shells from Risborough.

Vertical scale as in Fig. 2. One unit of horizontal scale = 180°.

and other sections may be treated in a similar way. Since all the measures in any section are known to be 180° apart, the position of one, relatively to the plane of reference, determines that of all the others.

By the means described, the position of every radius measured, with reference to the standard plane, was determined: and the length of each radius, in its proper position, is shown in Fig. 2, where the ordinate of every dot gives the length, and the abscissa the distance from the standard plane, of a single radius. The whole number of dots in the figure should be 1949, since this was the whole number of radii measured in the hundred shells: but many of the measures coincide in a diagram of this scale, so that the actual number of dots visible is smaller.

From these observations it was possible to determine a curve, representing a fairly close approximation to the law connecting change in the radius of the peripheral spiral with change in angular distance from the standard columellar radius. The determination was attempted in the following way. The observations were first sorted into groups, such that no two observations in any group differed by 180° in angular distance from the standard plane; the "centroid" of each group,—the point whose ordinate is the mean length of all the radii in the group, while its abscissa is the mean angular distance of all these radii from the standard plane—was determined, and this point was considered to lie upon the curve required. An inspection of Fig. 3 shows that the curve required is uniformly convex to the base line, so that the centroid of any segment lies in reality outside and above it. Small portions of the curve are however so nearly straight that the error introduced by the process adopted is not serious, at least in that portion of the curve which represents the upper part of the shell.

For reasons which will presently be pointed out, I have not determined the probable error of the mean radius length in every group, but only in six of them. The mean radius length of every group, together with the probable error of the six groups for which it was determined, is given in the following table:

TABLE I.

Mean Length of Peripheral Radius and Mean Angular Distance from the Columellar Plane, for each group of measures of 100 *adult Clausilia laminata.* (*Every mean value determined from* 100 *observations, unless otherwise stated.*)

(Angular distances *above* the plane of the standard columellar radius, towards the apex of the shell, are called negative.)

Angular Distance from Standard Columellar Radius	Mean Radial Length (mm.)
−11·4148 ×180°	0·4595 (37 observations)
−10·4906 ×180°	0·6254 (98 „)
− 9·50528×180°	0·8807
− 8·50528×180°	1·1237
− 7·50528×180°	1·3873
− 6·50528×180°	1·6813
− 5·50528×180°	1·9978
− 4·50528×180°	2·3443 ± 0·00815 mm.
− 3·50528×180°	2·7314 ± 0·00902 mm.
− 2·50528×180°	3·1504 ± 0·00905 mm.
− 1·50528×180°	3·6238 ± 0·00986 mm.
− 0·50528×180°	4·1366 ± 0·01054 mm.
+ 0·49472×180°	4·7206 ± 0·01147 mm.
+ 1·49472×180°	5·3599
+ 2·49472×180°	6·0885
+ 3·49472×180°	6·9119
+ 4·49472×180°	7·8879
+ 5·49472×180°	9·0029
+ 6·49472×180°	10·3057
+ 7·47936×180°	11·6877 (84 observations)
+ 8·56985×180°	12·4938 (26 „)
+ 9·66845×180°	13·5175 (4 „)

The regularity of the values obtained for the mean peripheral radius of the adult spiral made it seem that even so small a number of individuals might serve as a basis for comparison between young and mature individuals of the race. The way in which these shells are formed makes it less difficult to compare the mean magnitude and the variability of a character in young and in mature shells than it is in the case of most animal organs. For it is well known that the outer layers of the shell in *Clausilia* and other pulmonates are secreted by the growing edge of the mantle once for all; and that when the mantle edge has secreted these outer layers, the only subsequent change which can be effected is due to the deposition or absorption of the deepest layer, involving change in the thickness of the shell, but no change in its external shape. The only changes which the outer layers

can undergo are those due to erosion by weather, or to such absorption of the supporting inner layer as may cause some portions of the shell to be broken away from the rest. Neither of these disturbing influences affects *Clausilia laminata* in Gremsmühlen to any sensible extent: and we may therefore conclude that the upper whorls of an adult shell represent the condition of the young shell, from which this adult was formed by the addition of new material, in a practically unaltered form. This conclusion may not justify us in comparing the very last, newly formed portion of a growing shell with the corresponding part of an adult, but except at this point, there is no doubt of its validity.

Such an unaltered record of the young condition of the shell, borne by every adult examined, is of great value as a means of estimating the incidence of natural selection: for by measuring the mean and the standard deviation of radii of the spiral in the upper part of a series of adult shells, we obtain a measure of the mean length and the variability of the radii exhibited by these shells when they were still young; and in this way we can estimate the condition of the spiral in a series of young individuals, of which we can say with certainty that every one was capable of attaining the adult condition. In a sufficiently large sample of young shells, we are sure that we have a number of individuals which will die before they attain maturity; and therefore a comparison between the mean character and variability of these young shells, and the mean character and variability of the upper whorls of adult shells, enables us to determine whether there is a correlation between the character of the spiral, and the death-rate, or not. For if the death-rate during growth affects individuals with all sorts of spirals to the same extent, then there will be no change in the mean character and variability of the spiral after the time of its first formation, whether we examine it in half-grown individuals, soon after it has been formed, or in adults. If, on the other hand, individuals with different kinds of spirals die at different rates during growth, then either the mean length or the standard deviation of radii, or both, will change: so that we shall certainly not get the same value for *both* these characters in young and in adults, if the character of the spiral is being acted on by any process of selective elimination.

In order to compare the radii of young shells with those of the upper whorls of adults, one hundred half-grown individuals from Gremsmühlen were ground and measured, the measures being treated in the manner already described. The smallest of these individuals was 5·3 mm. long, the majority being from 6 to 7 mm. long. The length of the largest was 8·8 mm., but only three were more than 8 mm., the greater number being less than 7 mm. long, the mean length of an adult shell being about 15 mm.

The mean radius-length, with its mean angular distance from the plane of the standard columellar radius, is given for every group in Table II. The probable error of the mean radius-length has been determined for the six lowest groups, which contain the last-formed radii. These groups are those for which the probable error is given in adults also (see Table I.).

TABLE II.

Mean Length of Peripheral Radius, and Mean Angular Distance, of each group of measures in 100 *half-grown Clausilia laminata. (Every mean value determined from* 100 *observations, unless otherwise stated.)*

Mean Angular Distance from Standard Columellar Radius	Mean Length of Peripheral Radius
$-11{\cdot}31214 \times 180°$	0·4808 (50 observations)
$-10{\cdot}45494 \times 180°$	0·6482 (97 „)
$-9{\cdot}45872 \times 180°$	0·9132
$-8{\cdot}45872 \times 180°$	1·1446
$-7{\cdot}45872 \times 180°$	1·4154
$-6{\cdot}45872 \times 180°$	1·7021
$-5{\cdot}45872 \times 180°$	2·0367
$-4{\cdot}45872 \times 180°$	$2{\cdot}3770 \pm 0{\cdot}00950$
$-3{\cdot}45872 \times 180°$	$2{\cdot}7683 \pm 0{\cdot}01035$
$-2{\cdot}45872 \times 180°$	$3{\cdot}1838 \pm 0{\cdot}01043$
$-1{\cdot}45872 \times 180°$	$3{\cdot}6489 \pm 0{\cdot}01137$
$-0{\cdot}49560 \times 180°$	$4{\cdot}1457 \pm 0{\cdot}01205$ (91 observations)
$+0{\cdot}49048 \times 180°$	$4{\cdot}6952 \pm 0{\cdot}01489$ (62 „)

It is difficult to compare the values of the mean radii directly in young and in adults, because the mean angular distance is not the same in corresponding groups of measures. The mean angular distance is however so close to $\frac{180}{2}°$ in each case, that the value of the ordinate at that point, found by linear interpolation, will have a probable error only slightly greater than that of the entries in Tables I. and II. The necessary interpolations have been performed in Table III., the mean radius for the lowest (youngest) group of measures of the half-grown individuals being left unchanged, and the corresponding adult radius found by interpolation.

TABLE III.

Mean Peripheral Radius at Corresponding Points on the Spiral of 100 *Young and* 100 *Adult Clausilia.*

Angular Distance from Standard Columellar Radius in right angles	Mean Peripheral Radius	
	Adult	Young
−19	0·8820	0·9022
−17	1·1251	1·1350
−15	1·3889	1·4044
−13	1·6820	1·6903
−11	1·9996	2·0229
− 9	2·3463	2·3630
− 7	2·7336	2·7521
− 5	3·1529	3·1666
− 3	3·6265	3·6297
− 1	4·1397	4·1254
+0·9810	4·7181	4·6952

With probable errors of the order indicated by Tables I. and II., it is unlikely that any of these differences are significant. Even in the case of the last pair of entries the difference, although it is considerable (0·0229 mm.), is less than twice the probable error of the determination.

There is therefore no evidence of change in the mean character of the peripheral spiral during growth: on the contrary, the mean value of a peripheral radius is seen to be the same, within the limits of error of the determination, whether the radius be measured shortly after its formation, in young shells, or a long time afterwards, in the upper whorls of adult shells.

The individuals of the two samples measured may be considered, with sufficient accuracy, to represent samples of two successive generations; and the result obtained may be taken to show that the mean spiral of the young generation is sensibly identical with that of the parental generation, and is not altered during growth by any process of selective destruction. That is to say, individuals of *Clausilia laminata*, taken from a very old beech-wood, in a country which the species is known to have inhabited at all events since pre-glacial time, are in such equilibrium with their surroundings that the mean character of the shell spiral (so far as we have studied it) is the same from generation to generation, and is not changed during the course of a generation by any process of selective destruction.

If the view of inheritance, formulated by Mr Galton* and generalised by Professor Pearson†, be true, the mean character of a race can be fixed by natural selection in a far shorter time than is generally supposed; and we should be prepared to find local races in which the mean is sensibly stable through successive generations, in areas which have been inhabited even for a comparatively short period. It is, however, not necessary that the variability of such a race should be the same at all periods of growth. Professor Pearson has pointed out that if the law of inheritance referred to be approximately true, then a process of selection, so stringent that only individuals which accurately correspond to a particular type are allowed to breed, will fix this type as the mean of every generation after it has operated for a small number of generations; but even after such a selective process has operated for an indefinite time, the variability of young will be greater than that of the parental generation by an amount depending on the variability of the race at the time when selection began to operate. So that we may generally expect to find a reduction in the variability of a race during growth, long after selection has rendered the mean constant‡. Such selective destruction, which reduces the variability in every generation without changing its mean character, is what Professor Pearson calls "periodic selection."

In order to discover whether periodic selection is acting upon *Clausilia laminata* in Gremsmühlen or not, it is necessary to determine the variability of the

* Galton; *Roy. Soc. Proc.* Vol. 61, p. 402.

† Pearson; *Roy. Soc. Proc.* Vol. 62, p. 400.

‡ For a discussion of the residual variability here referred to, see Pearson; *Grammar of Science*, p. 384, and *Phil. Trans.* 1901 (in course of publication).

peripheral spiral at corresponding points of young and of adult shells. The proper measure of this variability is the standard deviation of the peripheral radii, in a number of individuals, measured at the same angular distance from the standard columellar radius, or the standard deviation of all the measures falling within a very narrow vertical strip of the diagram, Fig. 2. It would be impossible to find this standard deviation directly, without measuring many thousands of individuals; and the labour involved in preparing even a small series of measures is so great that a sufficiently large series for the purpose seems unattainable. Some indirect way of comparing the variability of young and of adults must therefore be found.

Before describing the method adopted in comparing the variability in young and in mature shells, a remark must be made. The variability of different parts of the peripheral spiral, as it appears from the distribution of the dots on Fig. 2, differs greatly in different regions. It is smallest in the immediate neighbourhood of the standard columellar radius, and increases as we pass away from this in either direction. The variability here indicated is not the variability of radii measured in different individuals at constant angular distance from the origin of the peripheral spiral: on the contrary, it is largely determined by the correlation between the different parts of the peripheral spiral and those radii of the columellar spiral which are nearly 5 mm. in length. The relative variability would be largely altered if we chose another columellar radius by which to determine the plane of reference. The artificial character of the variability which is here exhibited does not prevent us from using it to determine the probable error of the values obtained, or to establish a comparison between two sets of individuals which have been treated in exactly the same way.

If we consider any group of measures, such as those lying between two of the vertical divisions of Fig. 2, the straight line which gives the best linear approximation to their distribution is

$$y - y_0 = \frac{r_{xy}\sigma_y}{\sigma_x}(x - x_0),$$

where y_0 is the mean radial length in the group, σ_y the standard deviation of radial lengths; x_0 is the mean angular distance of the group from the standard columellar radius, σ_x the standard deviation of angular distances, and r_{xy} is Galton's function, measuring the correlation between radial length and angular distance from the standard radius*. The standard deviation of an "array" of radii within the group, whose angular distance is constant, is $\sigma_y\sqrt{1 - r^2_{xy}}$; and this will be taken as the measure of the variability of spiral radii at any point.

The values of r_{xy}, σ_y, and $\sigma_y\sqrt{1 - r^2_{xy}}$ for the six lowest groups of radii measured in the young shells, and for the corresponding groups of radii in adults, are given in Table IV. I wish here to express my gratitude to Miss Alice Lee, D.Sc., who has generously spent much time and labour in checking the whole

* Cf. Yule; *Roy. Soc. Proc.* Vol. 60, pp. 477—489.

of the computations connected with these Tables. Miss Lee has corrected several important arithmetical blunders of mine, and without her kind help I should certainly not have been able to publish this paper for some time. The results which follow from a study of this table seem to me of great importance; and such credit as may be due for obtaining them belongs far more to Miss Lee than to myself.

TABLE IV.

Values of r_{xy}, σ_y, and $\sigma_y\sqrt{1-r^2_{xy}}$ for corresponding groups of Peripheral Radii in Young and in Adult-shells from Gremsmühlen.

Angular Distance from Columellar Radius in right angles	r_{xy}		σ_y		$\sigma_y\sqrt{1-r^2_{xy}}$	
	Adult	Young	Adult	Young	Adult	Young
−8 to −10	0·7527 ±0·0292	0·7922 ±0·0251	0·120849	0·140872	0·07956 ±0·00379	0·08597 ±0·00412
−6 to −8	0·8068 ±0·0235	0·8374 ±0·0202	0·133731	0·153473	0·07901 ±0·00377	0·08388 ±0·00402
−4 to −6	0·8534 ±0·0183	0·8622 ±0·0173	0·134119	0·154627	0·06991 ±0·00333	0·07833 ±0·00375
−2 to −4	0·8726 ±0·0161	0·8782 ±0·0154	0·146252	0·168564	0·07143 ±0·00341	0·08062 ±0·00386
0 to −2	0·9290 ±0·0092	0·8800 ±0·0159	0·156270	0·170469	0·05784 ±0·00276	0·08096 ±0·00388
0 to 2	0·9415 ±0·0077	0·9367 ±0·0105	0·170046	0·173881	0·05732 ±0·00273	0·06087 ±0·00292

From Table IV. we see that the standard deviation of peripheral radii in the group from 8 to 10 right-angles from the standard columellar radius is nearly the same, both in young and in adult individuals—the small difference actually observed being hardly greater than the probable error of the determination. At every point, however, the variability of young shells is greater than that of adults; during the six right-angles of revolution immediately above the columellar radius the excess of variability in the young is always from about two to more than four times as great as the probable error of the determination.

The chances against an apparent excess of variability in a sample of young shells, so large as that recorded in the table, are very great, unless we admit that there is a real difference in variability between the newly-formed whorls of growing shells and the corresponding whorls of adults; and the necessary consequence of such an admission is that the variability of these newly-formed whorls is reduced after their formation by a process which destroys individuals with abnormal shells more rapidly than others, so that a process of "periodic selection" occurs.

If the result indicated by the observations we have discussed were true, without any qualification, then we might expect the variability of the very last-formed portion of a growing shell to be greater than that of any portion which precedes it; but the last line of Table IV. does not give the impression that this is the case. Of the young shells measured, 48 were so young that the last measurable part of their spire occurred above the point at which the columellar radius is 5 mm. long; but in 62 shells it was possible to measure a radius below this point: and the variability of these 62 radii appears, from the last line of Table IV., to be hardly, if at all, greater than that of the corresponding radii of adult shells.

The evidence of this small group of measures does not seem to me sufficient to outweigh that of the rest, because on the one hand the probable error of the determination is relatively large and on the other hand, an explanation of the apparent contradiction between the two sets of evidence may very possibly be found in some property of the last whorl of a growing shell. There are two obvious differences between the last-formed whorl of such a shell, and those which precede it. The last whorl is quite sensibly thinner than the others, because the inner, "nacreous" layer is not yet secreted in such quantity as in the higher parts of the shell; and the last whorl has nothing attached to its lower surface. Now, in the formation of a further whorl, the lower surface of the last-formed whorl will have a new layer of shell closely appressed against it: and my friend Professor Pearson has suggested to me that the strain on the shell, during this process, may cause a sensible increase in its variability. I think it very likely that some explanation of this kind will be found after further study of the last-formed whorl, which I hope shortly to undertake. In the meantime I publish the observations as they stand. They seem to me to show clearly that the mean character of the spiral radius is stable from generation to generation in Gremsmühlen, and that it is not being changed by selection during the growth of a young generation: and they seem to me to show further that the variability of young shells is diminished in every generation by a process of "periodic selection." Such selection is, of course, "indirect": that is to say, the life or death of the individual is determined in each case by the value of a (probably large) number of correlated characters, of which the length of the peripheral radius is only one.

I am aware that to many people it will seem absurd that the life or death of an individual should be supposed to depend upon differences of structural character so slight as those just described. The species of *Clausilia* show, however, if possible more clearly than other species, the way in which small and apparently unimportant differences of structure are associated with the difference between the survival and the total extinction of a race in a particular locality: and if so great a difference in death-rate as this is correlated with the apparently trifling differences which separate species, the hypothesis suggested by measurement of the spiral does not seem to me ridiculous.

At the risk of wearying the reader by repeating notorious facts, I wish to illustrate the relation between geographical distribution and specific characters by reference to a country which I know better than I know Holstein: and I therefore publish, in Table V., the mean lengths of the peripheral spiral radii in the upper whorls of a small series of adult *C. laminata* from a beech-wood in the neighbourhood of Monk's Risborough, in Buckinghamshire.

TABLE V.

Peripheral Radii of 32 *Adult Clausilia laminata from Risborough.*

Angular Distance from plane of Standard Columellar Radius in right angles	Mean Peripheral Radius
−19	0·8967
−17	1·1318
−15	1·4036
−13	1·6888
−11	2·0240
− 9	2·3686
− 7	2·7685
− 5	3·1741
− 3	3·6576
− 1	4·1395

The measurements of this race are not yet completed, and it is not worth while to discuss the probable errors of the small number of measures available; but a comparison between the results so far obtained and those recorded in Table III. (see also Fig. 3, p. 114) gives the impression that the races at Gremsmühlen and Risborough are sensibly identical in the mean character of their peripheral spiral.

The environmental elements common to the beech-wood near Monk's Risborough, where these shells were found, and the wood at Gremsmühlen, are those directly connected with the presence of beech-trees, and apparently few others. The wood (or copse) at Risborough is 800 feet above the sea level, on the steep side of a chalk down: it is of very small extent, and the beech-trees are tiny and young compared with the magnificent old trees at Gremsmühlen. The dryness of the ground is shown by the total absence of *Succinea*; while a difference of conditions important to terrestrial molluscs is shown by the abundance at Risborough of *Cyclostoma elegans*, a species which cannot live in the sandy plain of North Germany. The species of *Helix*, mentioned as common in Gremsmühlen, also occur in the wood at Risborough, but the prevalent colour varieties of *H. nemoralis* are different, and there are well-marked differences between the races of *H. arbustorum* in the two localities. In addition to these species, *H. aspersa*, which does not occur east of the Rhine, and *H. cantiana*,

which occurs round Hamburg, are common at Risborough, but absent from Gremsmühlen. The small size of the Risborough wood, and the consequent absence of true forest conditions, permit the free entrance of *H. variabilis*. The only species of *Clausilia* which occur are *C. laminata* and *C. nigricans*.

The species of *Helix* mentioned, and *Clausilia nigricans*, are found in the hedges or meadows which surround the little wood at Risborough, but *Clausilia laminata* occurs only in the beech-wood. A similar relation between *Clausilia laminata* and *C. nigricans* can be demonstrated on many of the English chalk downs. In the places where beech-trees occur, the two species are found together; but *C. laminata* does not occur, or occurs but rarely, away from beech-trees, while *C. nigricans* is found in other places also.

These two races of *C. laminata* show *first* that races with a sensibly identical spiral exist in localities so widely separated that no crossing between their ancestors can have occurred for an immense period of time, and so different that the environmental conditions common to both are comparatively few; *secondly* that the existence of *C. laminata* is possible under conditions which permit the existence of *C. nigricans*; but that the latter species can live and multiply in places where the conditions necessary for the life of *C. laminata* do not occur. Now the differences between these two species are (1) a difference in size; (2) a difference in the pitch of the spiral; and (3) other slight differences in the shape of the shell and of various organs. All these differences seem to us, at present, of little physiological importance, and yet the sum of these differences is demonstrably associated with an enormous difference in susceptibility to certain environmental differences: and since this is the case we are justified in accepting evidence which points to a correlation between variation in death-rate and variation in a character within the limits of one species, although we are at present quite unable to imagine the process by which this correlation is brought about.

MISCELLANEA.

I. Change in Organic Correlation of *Ficaria ranunculoides* during the Flowering Season.

Among the numerous statistical studies we owe to Professor Julius MacLeod is one which deals with the change in correlation between the number of stamens and the number of pistils in *Ficaria ranunculoides* during a single flowering season [Over de Correlatie tusschen het aantal meeldraden en het aantal stampers bij het Speenkruid (*Ficaria ranunculoides*): *Botanisch Jaarboek*, Jaargang XI. 1899]. The number of stamens, and the number of pistils, were determined in each of a series of flowers borne by certain plants at the beginning of the flowering season, and again in a series of flowers from the same plants at the end of the season. The correlation between the number of stamens and the number of pistils differed widely in the two cases. The observations were made on the same plants (or their asexual offspring) during two successive seasons, and similar results were obtained on each occasion. The coefficient of correlation was in each case lower at the beginning of the flowering season, higher at the end; so that the same plants are shown to exhibit a rhythmical change in the amount of correlation between two important sets of organs. This fact is of very great interest in its bearing upon our knowledge of organic correlation in general: and it is to be regretted that Professor MacLeod should have expressed his valuable result in a manner which fails to exhibit its full importance, through his adoption of Verschaffelt's method of stating correlation.

Dr Verschaffelt has always failed, so far as I am acquainted with his writings, to distinguish between correlation and regression, so that he is unable to find a single constant, expressing the correlation between two variables, except in very special cases. It also follows that his estimate of the amount of correlation is generally erroneous.

The notation introduced by Mr Galton, when he first used the conceptions of correlation and regression in biological work, enables us to express the important result obtained by Professor MacLeod with great ease: and in order to show this, the Tables relating to the flowers observed in 1899 are here reprinted.

Table II. shows the frequency of flowers with a given number of stamens and pistils among the "late" flowers, which appeared from April 17 to 23, 1899. The column at the extreme right of the Table gives the whole number of flowers with a given number of stamens, the line at the foot gives the whole number with a given number of pistils. From these we find that the mean number of stamens (M_s) was 17·863271, the mean number of pistils (M_p) was 12·147453 *. The Standard Deviation or Error of Mean Square of the number of stamens, which we will call σ_s, was 3·29840; that of the number of pistils (σ_p) was 3·38776. Now if we call Δ_p a deviation from the mean number of pistils, and Δ_s a deviation from the mean number of stamens, the best value of the coefficient of correlation is

$$r = \frac{S(\Delta_p \Delta_s)}{n \sigma_p \sigma_s},$$

where n is the number of observations, and $S(\Delta_p \Delta_s)$ is the sum of the products of all associated deviations†. The value of this expression, for Table II., is

$$r = \frac{3121{\cdot}520053}{4167{\cdot}97287} = 0{\cdot}7489.$$

* I have to thank Miss M. A. Lewenz, of University College, London, for going over the computations connected with these Tables.

† See Pearson; *Phil. Trans.* A. 1896; and Yule; *Roy. Soc. Proc.* LX.

TABLE I.

268 *Early Flowers.*

Number of Pistils

Number of Stamens	2	3–6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	Totals
18												1																1
19						1			1	4																		6
20								3	1	3	1																	8
21							2	1	1		2	2		1														9
22								2	4	1	4		2			2	1											16
23								1	1	2	3		2	2	1													12
24								1	1	4	7	1	1	4	2	1												22
25						1	1	3	1	1	4	5	4	3		1		1					1					26
26								1		2	4	3	2	1	5	2	2	2	1	1								26
27								1		3	5	5	3	7	4	2	2	3	1	1			1					38
28									1	1	2	4	1	1	4													14
29			1		1				1	1	1	5	2	3	1	1		1	2	2							1	23
30												3	5	2	1	3	2	2	1			1						20
31											1	2	2	1	1	4	4	3	1		1							20
32											1		1		1	2	1	1	2		2	2						13
33														2	1	1	1	2										7
34																								1				1
35																			2				2					4
36																												0
37	1																											1
38																					1							1
Totals	1	0	1	0	1	2	3	13	12	22	35	31	25	27	21	19	13	15	10	4	4	3	4	1	0	0	1	268

TABLE II.

373 *Late Flowers.*

Number of Pistils

Number of Stamens	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	Totals
8							1														1
9																					0
10		1		1	2																4
11	1	1	2					1	1												6
12			4		1																5
13		1	4	3	2	1															11
14			2	10	3	3	2	2	1												23
15		3	4	14	7	4	5	4			1	1									43
16				2	10	6	8	4	4			1									35
17				1	2	5	6	3	7	2	2	2	1								31
18				3	7	13	6	5	13	8	5	3	3								66
19				1		4	7	7	9	7	4	0	3		1						43
20					1	1	5	6	6	12	2	5	2	1	1						42
21								3	2	4	1	3	4	2							19
22										2	3	5	1	2	2	1		1			17
23									1		1	1		2	1		1				7
24						1					1	1			1						4
25												2	2		1					1	6
26												1		3	1				1		6
27									1							1					2
28															1						1
29														1							1
Totals	1	6	16	35	35	38	40	35	45	36	21	23	16	11	9	2	1	1	1	1	373

This is what Mr Galton has called a *coefficient of correlation*, and it must be distinguished from a *coefficient of regression*. In the case before us, if we take an "array" of flowers, such that the number of stamens in every flower differs from the mean number by Δ_s, then the mean number of pistils in the array will be

$$M_p + r\frac{\sigma_p}{\sigma_s}\Delta_s,$$

where $r\frac{\sigma_p}{\sigma_s}$ is the "coefficient of regression." Now in this case the quantity which Dr Verschaffelt proposes as the measure of correlation is, if I understand him rightly, the mean for all arrays of

$$\frac{r\frac{\sigma_p}{\sigma_s}\Delta_s}{\frac{M_p}{\Delta_s}} = r\frac{\sigma_p}{\sigma_s}\cdot\frac{M_s}{M_p};$$

and by using this value as a measure of correlation, Professor MacLeod has been led to state his interesting results in a rather erroneous way. In the first place, since Dr Verschaffelt's measure of correlation is really in part a function of a measure of regression, it has one value if we determine the mean number of pistils associated with a given deviation from the mean number of stamens, another if we start with known deviation from the mean of pistils, and proceed to determine the mean associated deviation of stamens: for the mean number of stamens in an array of deviation Δ_p from the mean number of pistils is

$$M_s + r\frac{\sigma_s}{\sigma_p}\Delta_p,$$

and Dr Verschaffelt's measure of correlation in this case is $r\frac{\sigma_s}{\sigma_p}\frac{M_p}{M_s}$. But, apart from this, the introduction of the ratio between the means of the two variables has so much effect as to destroy the value of the quantity proposed as a measure either of regression or of correlation. We see this in Professor MacLeod's treatment of his results.

If we treat Table I., based upon 268 "early" flowers, which appeared between February 27 and March 17, as we treated the table of "late" flowers, we find

$$M_s = 26{\cdot}731343\,; \qquad \sigma_s = 3{\cdot}76088$$
$$M_p = 17{\cdot}447761\,; \qquad \sigma_p = 3{\cdot}89425$$
$$r = 0{\cdot}5065,$$

so that the correlation between number of stamens and number of pistils is very distinctly less in early than in late flowers. The probable error of the value of r for early flowers is 0·0306, and for late flowers it is 0·0153; so that the difference between the observed values is certainly significant.

In an array of early flowers, all of which have a number of stamens differing from the mean number by Δ_s, the mean number of pistils should, from the values obtained, be

$$17{\cdot}447761 + \frac{0{\cdot}5065 \times 3{\cdot}89425}{3{\cdot}76088}\Delta_s$$
$$= 17{\cdot}447761 + 0{\cdot}524\,\Delta_s,$$

or a given deviation from the mean number of stamens should be associated with a deviation of little more than half that amount from the mean number of pistils.

An examination of Table I. will show that this conclusion is as nearly in accord with observation as the numbers in the table would lead one to expect. It is certainly far more intelligible than that to which we are led by Dr Verschaffelt's method, which induces Professor MacLeod to assert that the correlation is in this case nearly perfect,—the difference being due to Dr Verschaffelt's use of the ratio between the means, so that the "measure of correlation" obtained by his method should be $0{\cdot}524\frac{M_s}{M_p} = 0{\cdot}803$.

In the case of the late flowers, Dr Verschaffelt's measure of correlation gives a value *greater* than unity; and Prof. MacLeod seems to have taken this as a sign of imperfect correlation: so that he describes the change in correlation as the reverse of that which actually occurs, and to speak of stamens and pistils as less perfectly correlated in late flowers than in early ones.

Dr Verschaffelt's method of attempting to measure correlation is so extensively used by botanists in his own country that it has seemed worth while to point out in some detail the divergence between his method and that based by Mr Galton on the theory of chance, and at the same time to indicate how it may lead to erroneous conclusions.

Professor MacLeod's investigations on the differences in mean, variability and correlation of the same plants during the flowering season provide a most valuable lesson as to the possible danger of asserting that such differences are significant of local races. They may be due solely to the local environment, or to the period at which the individuals were collected; the seasonal changes in these characters may be accelerated or delayed by local conditions.

W. F. R. WELDON.

II. Statoblasts of *Pectinatella Magnifica.*

In an interesting paper on the variation of the statoblasts of this freshwater Bryozoa in *The American Naturalist*, Vol. XXXIV. p. 964, Professor Davenport has determined (*a*) the standard deviation, σ, of the number of hooks of the whole population of 635 statoblasts. He finds it to be 1·326. He has also found (*b*) the average standard deviation, Σ, of eleven colonies of statoblasts. Each such colony is derived from a single statoblast. He finds the average value 1·197. We thus have:

$$\frac{\Sigma}{\sigma} = \frac{1{\cdot}197}{1{\cdot}326} = {\cdot}9027.$$

Now a statoblast colony is one the ancestry of which has been selected back indefinitely. Hence if we accept the "Law of Ancestral Heredity"* we should expect that $\Sigma/\sigma = {\cdot}8944$, a result in good accordance with the above. Thus heredity in this simple organism seems quite comparable with its value as found for man and the higher mammals, an exceedingly important result. We may hope that measurements like those of Professor Davenport will be carried out more extensively and on other species, for there is no more interesting problem than that involved in the diversity or constancy of heredity throughout living forms.

K. P.

* *R. S. Proc.* Vol. 62, p. 399, Mr Yule points out to me that a deduction of the fraternal correlation from this result—such as I have given in my paper "On the Principle of Homotyposis and its relation to Heredity" (*Phil Trans.* A. 1901, see Table XXXI.)—is open to question.